

Math Finance, Homework 5, Winter 2006
Due Wednesday, February 8, 2006

1. Consider a Multi-Period model with $T = 3$.
 - (a) Compute $\mathbf{E}^* [\xi_3 | \mathcal{F}_2]$ by using the definition of conditional expected value.
 - (b) Show that ξ_3 is independent of \mathcal{F}_2 .
 - (c) Now use the fact that you proved in part 1b to compute $\mathbf{E}^* [\xi_3 | \mathcal{F}_2]$.
 - (d) Show that $\mathbf{E}^* [S_3^* | \mathcal{F}_2] = S_2^*$.

2. Fix a probability space (Ω, \mathbf{P}) such that Ω is a finite set, random variables Y and Z defined on (Ω, \mathbf{P}) , and a partition \mathcal{P} of Ω . Let \mathcal{F} be the σ -algebra generated by \mathcal{P} .
 - (a) Show that
$$\mathbf{E}[Y + Z | \mathcal{F}] = \mathbf{E}[Y | \mathcal{F}] + \mathbf{E}[Z | \mathcal{F}].$$
 - (b) Use the above linearity property to redo problem 4 from homework 4. For this note that if you let G_i be the net gain from game i for $i = 1, 2, 3$, then the net gain is the sum of the net gains from each game, i.e., $G = G_1 + G_2 + G_3$.