Math Finance, Homework 6, Winter 2006
Due Wednesday, February 15, 2006

1. Consider a T-Period binomial model. Let V be a random variable defined on $(\Omega_T, \mathbf{P})$ that is nonnegative. Show that $\mathbf{P}(V > 0) > 0$ if and only if $\mathbf{E}^*[V] > 0$.

2. Parts (a), (b), and (c) of exercise 2 in Section 2.4 of the Williams text. In part (a), compute arbitrage free price for time zero, one, and two.

3. Consider a 3-Period Binomial Model with $r = 1$, $d = 1$, $u = 3$, $p = 2/3$, and $S_0 = 20$. Let X be the value at time 3 of a European *put* option with strike price \$68 and expiration time 3.

   (a) Determine the arbitrage free price $\{C_t, t = 0, 1, 2\}$ for the put option.

   (b) Suppose that

   $$C_1 = \begin{cases} 6 & \text{on } \{\omega_5, \omega_6, \omega_7, \omega_8\}, \\ 4 & \text{on } \{\omega_1, \omega_2, \omega_3, \omega_4\}. \end{cases}$$

   Give an example of an arbitrage opporunity, and verify that your example is indeed an arbitrage opportunity.

4. (a) Suppose that $\{Y_t, t = 0, 1, \dots\}$ is a $\mathbf{P}^*$-martingale . Show that for all $0 \le s < t \le T$,

   $$\mathbf{E}^*[Y_t \mid \mathcal{F}_s] = Y_s.$$

   Conclude that $\mathbf{E}^*[Y_t] = Y_0$.

   (b) Suppose that ψ is a self-financing trading strategy in the secondary market and that $\{C_t^*, t = 0, 1, \dots, T\}$ is a $\mathbf{P}^*$-martingale. Show that $\{V_t^*(\psi), t = 0, \dots, T\}$ is a $\mathbf{P}^*$-martingale.

   (c) Use the results of parts (a) and (b) to give an alternative proof that $C_t = V_t(\phi^*)$ $t = 0, 1, \dots, T$ is arbitrage free.