

**Math Finance, Homework 7, Winter 2006**  
**Due Wednesday, February 22, 2006**

1. For an American contingent claim, verify the following.

(a) For  $t = 1, \dots, T$ , the money required at time  $t - 1$  to replicate  $U_t$  is

$$\frac{\mathbf{E}^* [U_t | \mathcal{F}_{t-1}]}{1 + r}.$$

That is show that  $\mathbf{E}^* [U_t^* | \mathcal{F}_{t-1}] = \tilde{\alpha}_t S_{t-1}^* + \tilde{\beta}_t$ .<sup>1</sup>

(b) Suppose that  $t$  is such that  $1 \leq t \leq T$  and  $\tilde{\phi}_t = \phi_t^*$ . Show that the money required at time  $t - 1$  to replicate  $U_t$  is  $V_{t-1}(\phi^*)$ .<sup>2</sup>

2. Consider model inputs from Exercise 2 in Section 2.4 of the text.

(a) In a previous homework, you found the replicating strategy for the European call option. Check your work by using the manufacturing cost lemma and make the appropriate corrections.<sup>3</sup>

(b) Find the superhedging strategy for the American call option and the time zero manufacturing cost.<sup>4</sup>

3. Consider the model inputs from Exercise 6 in Section 2.4 of the text.

(a) Find the replicating strategy for the European put option and the time zero manufacturing cost.<sup>5</sup>

(b) Find the superhedging strategy for the American put option and the time zero manufacturing cost.<sup>6</sup>

4. Compare your results in 2 and 3. How do the results differ?

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<sup>1</sup>Hint: Express  $U_t^*$  in terms of  $\tilde{\phi}_t$  and  $S_t^*$ . Also, the risk neutral property may come in handy.

<sup>2</sup>Combine the given information with what you proved in part (a).

<sup>3</sup>Doing this will make part (b) much easier.

<sup>4</sup>Check your work at each stage by using the appropriate result in problem 1.

<sup>5</sup>Remember to check your work at each stage by using the manufacturing cost lemma.

<sup>6</sup>Check your work at each stage by using the appropriate result in problem 1.