(1) Write your Name and PID on the front of your exam sheet.
(2) No calculators or other electronic devices are allowed during this exam.
(3) Show all of your work; no credit will be given for unsupported answers.
(4) Read each question carefully to avoid spending your time on something that you are not supposed to (re)prove.
(5) Ask me when you are unsure if you are allowed to use certain fact or not.

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<thead>
<tr>
<th>Problem</th>
<th>Score out of 10</th>
</tr>
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<tbody>
<tr>
<td>1</td>
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<td>Total out of 30</td>
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(1) Which one of the following propositional forms is NOT equivalent to $P \Rightarrow (Q \lor R)$? Justify your answer.

(a) $(P \land (\neg Q) \land (\neg R)) \Rightarrow O$.
(b) $(P \land (\neg Q)) \Rightarrow R$.
(c) $(P \Rightarrow Q) \land (P \Rightarrow R)$.
(d) $(\neg P) \lor Q \lor R$.

(You have to only prove why your chosen propositional form is not equivalent to $P \Rightarrow (Q \lor R)$. You do NOT need to argue why the rest are equivalent.)
(2) Prove that there are no integers $m$ and $n$ such that $7m + 21n = 15$. 
(3) Let $a_1 = 1$ and

$$a_{n+1} = 1 + \frac{1}{1 + \frac{1}{a_n}}$$

for any positive integer $n$. Prove that for any positive integer $n$ we have that $a_n < a_{n+1}$. (Hint: if $0 < b < c$, then $\frac{1}{c} < \frac{1}{b}$. And you can assume $a_n > 0$ without proof.)