

1. Construct truth tables for the following propositional forms.

(i)  $P \wedge (Q \vee R)$

(iii)  $\neg(P \wedge Q)$

(v)  $\neg P \vee Q$

(ii)  $(P \wedge Q) \vee (P \wedge R)$

(iv)  $(\neg P) \vee (\neg Q)$

(vi)  $P \Rightarrow Q$

2. Use the first problem to deduce that for any propositional forms  $P$ ,  $Q$  and  $R$ ,

(i)  $P \wedge (Q \vee R)$  and  $(P \wedge Q) \vee (P \wedge R)$  are equivalent.

(Distributive law.)

(ii)  $\neg(P \wedge Q)$  and  $(\neg P) \vee (\neg Q)$  are equivalent.

(DeMorgan's law)

(iii)  $P \Rightarrow Q$  and  $\neg Q \Rightarrow \neg P$  are equivalent.

( $\neg Q \Rightarrow \neg P$  is called the contrapositive of  $P \Rightarrow Q$ .)

3. Explain why "This sentence is false" is NOT a proposition.

4. Complete the following truth table.

$x$	$y$	$x < y$	$x = y$	$x \leq y$
1	1			
1	0			
0	1			

5. Prove that  $|x+y| \leq |x| + |y|$  for every real numbers  $x$  and  $y$ .

6. Find a propositional form expressed only in terms of  $P, Q, R, \wedge, \vee, \neg, (, )$  whose truth table is the following:

P	Q	R	*
T	T	T	F
T	T	F	T $\rightsquigarrow$ ①
T	F	T	F
T	F	F	T $\rightsquigarrow$ ②
F	T	T	F
F	T	F	T $\rightsquigarrow$ ③
F	F	T	F
F	F	F	F

(Hint: \* is true if and only if ① holds or ② holds or ③ holds  
 ① holds if and only if  $P \wedge Q \wedge \neg R$  is true.  
 ... )