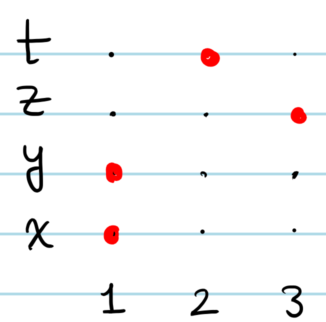
1. In each case, determine if the given set is the graph of a function. If it is the graph of a function f, say what is the domain and the image of f. Justify your answer.

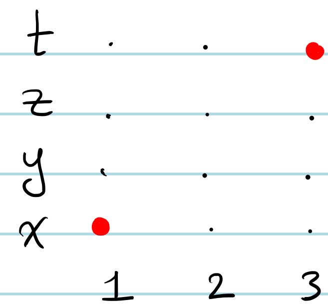
(a) $\{(1,1),(2,1),(3,4)\}$

(b) $\{(1,1),(1,2),(4,3)\}$

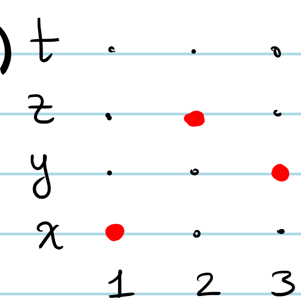
(c)



(d)



(e)



(In parts (c), (d) and (e), the domain of f is $\{1,2,3\}$ and the codomain is $\{x,y,z,t\}$.)

2. Let A and B be two non-empty sets and $f: A \to B$ be a function. Prove that f is one-to-one if there exists a function $g: B \to A$ such that $g \circ f = I_A$.

3. Let A and B be two non-empty sets and $f: A \to B$ be a function. Prove that f is onto if there is a function $g: B \to A$ such that $f \circ g = I_B$.

4. **(Bonus Problem)** Prove the converse of problems 2 and 3.

5. Let $A \subseteq X$ be two non-empty sets. Is there a function $f: X \to X$ such that $\mathrm{Im}(f) = A$? Justify your answer.

(<u>Hint</u>: Use the same idea as the example presented in class where we gave a function $f: \mathbb{R} \to \mathbb{R}$ s.t. $\mathrm{Im}(f) = \mathbb{R} \setminus \mathbb{Z}$.)

6. Let A, B and C be three non-empty sets and $f: A \to B$ and $g: B \to C$ be two functions.

(a) Prove that, if f and g are injective, then $g \circ f$ is injective.

(b) Prove that, if f and g are surjective, then $g \circ f$ is surjective.

7. Is there a function $f: (0,1) \to \mathbb{R}$ which is a bijection?

(You are allowed to use whatever you need from calculus without proof. For instance you are allowed to use sketch of the graph of a famous function or mean value theorem.)