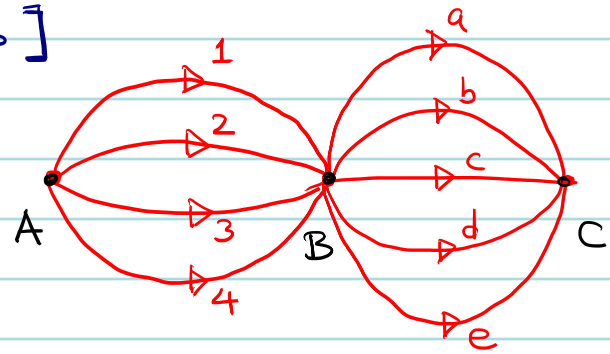
1. In how many ways can one go from A to C?

[(1) We have to follow the arrows]



2. Use the multiplication principle to prove that, if $|A_i| = m_i$ for $1 \le i \le k$, then $|A_1 \times A_2 \times \cdots \times A_k| = m_1 \cdot \ldots \cdot m_k$.

[Hint: Prove that $f : (A_1 \times A_2 \times \cdots \times A_k) \times A_{k+1} \longrightarrow A_1 \times \cdots \times A_{k+1}$

$$f((a_1, \ldots, a_k), a_{k+1}) := (a_1, a_2, \ldots, a_{k+1})$$

is a bijection and prove the desired result by induction on k.]

3. How many functions are there from $A = \{1, 2\}$ to $B = \{a, b, c, d, e\}$? [Hint: Show that there is a correspondence between the set

$$\text{Fun}(A, B) := \{f \mid f : A \to B \text{ is a function}\}$$

of functions from A to B and $B \times B$.

(Consider $f \longmapsto (f(1), f(2))$.)]

4. How many functions are there from $A=\{1,2,3,\dots,n\}$ to $B=\{0,1\}$? [Hint: Show that there is a correspondence between the set $\mathrm{Fun}(A,B)$ of functions from A to B and $\underbrace{B\times B\times\cdots\times B}_{n \text{ copies}} = \{(a_1,\dots,a_n) \mid \forall i;\ a_i=0 \text{ or } a_i=1\}$.]

5. Let $X=\{n\in\mathbb{Z} \mid 1\le n\le 300\}$,

$A=\{n\in X \mid 2\mid n\}$,

$B=\{n\in X \mid 3\mid n\}$,

$C=\{n\in X \mid 5\mid n\}$.

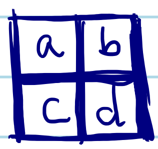
Find $|X\setminus(A\cup B\cup C)|$.

(Hint: ① $|\{k\in\mathbb{Z} \mid 1\le k\le n,\ d\mid k\}| = n/d$ if $d\mid n$ and $d,n\in\mathbb{Z}^+$.

② $2\mid n \wedge 3\mid n \iff 6\mid n$
$2\mid n \wedge 5\mid n \iff 10\mid n$
$3\mid n \wedge 5\mid n \iff 15\mid n$
$2\mid n \wedge 3\mid n \wedge 5\mid n \iff 30\mid n$.)

6. Let P_1, P_2, P_3, P_4 and P_5 be five points is a unit square.

Prove $|P_iP_j| \le 1/\sqrt{2}$ for some $i\neq j$.

(Hint: ① Pigeonhole principle. ② 

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