1. \begin{array}{|c|c|c|c|c|c|c|c|}
\hline
P & Q & R & Q \lor R & P \land Q & P \land R & P \land (Q \lor R) & (P \land Q) \lor (P \land R) \\
\hline
T & T & T & T & T & T & T & T \\
T & T & F & T & T & F & T & T \\
T & F & T & F & T & F & T & T \\
T & F & F & F & F & F & F & F \\
F & T & T & T & F & F & F & F \\
F & T & F & T & F & F & F & F \\
F & F & T & T & F & F & F & F \\
F & F & F & F & F & F & F & F \\
\hline
\end{array}

2. (I) and (II) imply that \((P \land (Q \lor R)) \equiv (P \land Q) \lor (P \land R)\).

III and IV imply that \((- (P \land Q)) \equiv (-P) \lor (-Q)\).

V and VI imply that \((P \Rightarrow Q) \equiv (-P) \lor Q\).

We use VII to deduce the above equivalency.
3. This is NOT a proposition as we cannot assign a truth-value to it.

- If it is true, then it says it is false which is a contradiction.
- If it is false, then it is NOT false which is a contradiction.

4. \[ x \quad y \quad x < y \quad x = y \quad x \leq y \]

<table>
<thead>
<tr>
<th></th>
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<th>T</th>
<th>T</th>
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</thead>
<tbody>
<tr>
<td>1</td>
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<td>F</td>
<td>T</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
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<tr>
<td>0</td>
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</tbody>
</table>

5. \[ x \geq 0 \quad y \geq 0 \quad x + y \geq 0 \quad |x| \quad |y| \quad |x + y| \quad |x| + |y| - |x + y| \quad |x| + |y| - |x + y| \]

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<th>T</th>
<th>0</th>
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<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>x-y</td>
<td>x+y</td>
<td>-2x-2y</td>
<td>F</td>
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<tr>
<td>T</td>
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<td>T</td>
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<td>x-y</td>
<td>x+y</td>
<td>-2y</td>
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<td>-x-y</td>
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</tbody>
</table>

To get the last column, we look at the value of
$|x| + |y| - |x+y|$ and find out if it is non-negative. For instance, when it is 0, it is non-negative.

The third entry: it is $-2y$. In that row, the truth-value of $y \geq 0$ is false. So $y < 0$.

Hence $-2y > 0$.

In order to show $|x| + |y| \geq |x+y|$, it is enough to say why the second and the seventh cannot happen.

_Second row:_ $x \geq 0$ and $y \geq 0$ imply $x+y \geq 0$. So the second row cannot happen.

_Seventh row:_ $x < 0$ and $y < 0$ imply $x+y < 0$. Hence the seventh row cannot happen.

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>R</th>
<th>( f(P, Q, R) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
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</tbody>
</table>

\( f(P, Q, R) \) is true $\iff$ either \( P \land Q \land \neg R \) is true or \( P \land \neg Q \land \neg R \) is true or \( \neg P \land Q \land \neg R \) is true.
Hence \( f(P,Q,R) \) is true \iff \( (P \land Q \land \neg R) \lor (P \land \neg Q \land \neg R) \lor (\neg P \land Q \land \neg R) \) is true.

Therefore
\[
f(P,Q,R) \equiv (P \land Q \land \neg R) \lor (P \land \neg Q \land \neg R) \lor (\neg P \land Q \land \neg R)
\]

[Students might give a different propositional form (which is equivalent to what I have written here). Of course, it is OK. The advantage of the above solution is the fact that it is algorithmic and shows that any propositional form is equivalent to a propositional form involving only \( \land, \lor, \neg \) and (, ).]

After simplifying, one can show that
\[
f(P,Q,R) \equiv (R) \land (P \lor Q)
\]

[5. (Alternative)]

\[xy \leq |xy| \implies x^2 + 2xy + y^2 \leq |x|^2 + 2 |xy| + |y|^2 \]
\[\implies (x+y)^2 \leq (|x|+|y|)^2\]
\[\begin{align*}
\text{②} & \quad |x+y|^2 \leq (|x|+|y|)^2 \\
\text{③} & \quad \Rightarrow |x+y| \leq |x|+|y|.
\end{align*}\]

(Clearly this proof is constructed backwards.)

In the above argument we are using the following facts:

① \quad a \leq |a| \text{ for any real number } a.

Proof of ① If \(a \geq 0\), then \(|a| = a\).
\[\text{If } a < 0, \text{ then } |a| = -a > 0 > a. \]

② \quad a^2 = |a|^2 \text{ for any real number } a.

Proof of ② If \(a \geq 0\), then \(|a| = a \Rightarrow |a|^2 = a^2\).
\[\text{If } a < 0, \text{ then } |a| = -a \Rightarrow |a|^2 = a^2. \]

③ \quad If \(a\) and \(b\) are non-negative real numbers and \(a^2 \leq b^2\), then \(a \leq b\).

Proof of ③ If not, \(a > b \geq 0\). Thus \(a^2 > ab \geq b^2\).
which is a contradiction. \[\]

[5. (Alternative)]

\(|a| = a \text{ if } a \geq 0 \text{ and } -a \text{ if } a < 0\).
\[ \Rightarrow \begin{cases} \pm x \leq |x| \\ \pm y \leq |y| \end{cases} \Rightarrow \pm (x+y) \leq |x|+|y| \\
\Rightarrow |x+y| \leq |x|+|y| \cdot \]