Second problem set

1. ⓐ Find all the solutions of x^2-x-2 in $\mathbb{Z}_{17}$.

   ⓑ Does x^2-x-2 have only two zeros in $\mathbb{Z}_{18}$?

2. Show that the characteristic of an integral domain is either zero or prime.

3. Find the characteristic of $\mathbb{Z}_4 \times \mathbb{Z}_6$ and $\mathbb{Z}_6 \times \mathbb{Z}_8 \times \mathbb{Z}_9$. (Justify your answer.).

4. Let R be a ring that contains at least one non-zero element. Suppose, for any $x \in R$, there is a <u>unique</u> $y \in R$ such that $xyx = x$.

   ⓐ Prove that R has no zero divisors.

   (<u>Hint</u>. If $xx' = 0$, then consider $x(y+x')x$.)
   $\phantom{(\text{Hint}.\ }$ or $x'x = 0$

   ⓑ Prove that $yxy = y$. (<u>Hint</u>. Consider $x(yxy)x$.)

   ⓒ Prove that R has a unity.

   (<u>Hint</u>. Consider $(xyx)z = (x)z$ and $z(yxy) = z(y)$. And use the cancellation laws.)

   ⓓ Prove that R is a division ring.