QUIZ 3, VERSION A, MATH103A, SUMMER 2021

1. Determine if the following statements are true or false. Briefly justify your answer.
   (a) (2 points) In $\mathbb{Z}_5^\times$, $o([2]_5) = 4$.

   (b) (2 points) $(\mathbb{R}^+, \cdot) \simeq (\mathbb{R}, +)$ where $\mathbb{R}^+$ is the set of positive real numbers.

   (c) (2 points) Every element of S_7 has order at most 7.

   (d) (2 points) In a group G, if $g_1^2 = g_2^2 = e_G$, then $(g_1 g_2)^2 = e_G$.

2. Suppose $G = \langle g \rangle$ is a group of order 30.
   (a) (2 points) Notice that $G \times G$ is a group under the following multiplication:
   $$(x_1, x_2) \cdot (y_1, y_2) = (x_1 \cdot y_1, x_2 \cdot y_2).$$
   Show that for every $(x, y) \in G \times G$, we have $(x, y)^{30} = (e_G, e_G)$.

   (b) (3 points) Prove that $G \times G$ is not a cyclic group.

   (c) (2 points) Suppose $o(g^k) = 15$. Find $\gcd(k, 30)$.

   (d) (3 points) How many elements of G have order 15?

   (e) (2 points) How many subgroups does G have?

3. Suppose $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 10 & 9 & 7 & 8 & 1 & 3 & 2 & 4 & 6 & 5 \end{pmatrix}$.
   (a) (3 points) Find a cycle decomposition of σ.

   (b) (2 points) Find $|\langle \sigma \rangle|$.

   (c) (3 points) Find a cycle decomposition of σ^{59}.

   (d) (2 points) Is σ odd or even? Justify your answer.