QUIZ 3, VERSION B, MATH103A, SUMMER 2021

1. Determine if the following statements are true or false. Briefly justify your answer.
   (a) (2 points) In $\mathbb{Z}_9^{\times}$, $o([2]_9) = 6$.

   (b) (2 points) $(\mathbb{R} \setminus \{0\}, \cdot) \simeq (\mathbb{R}, +)$ where $\mathbb{R} \setminus \{0\}$ is the set of non-zero real numbers.

   (c) (2 points) There is no element of S_9 that has order 20.

   (d) (2 points) In S_3, there are transpositions τ_1 and τ_2 such that $o(\tau_1 \tau_2) = 3$.

2. Suppose $G = \langle g \rangle$ is a group of order 70.
   (a) (2 points) Notice that $G \times G$ is a group under the following multiplication:
   $$(x_1, x_2) \cdot (y_1, y_2) = (x_1 \cdot y_1, x_2 \cdot y_2).$$
   Show that for every $(x, y) \in G \times G$, we have $(x, y)^{70} = (e_G, e_G)$.

   (b) (3 points) Prove that $G \times G$ is not a cyclic group.

   (c) (2 points) Suppose $o(g^k) = 14$. Find $\gcd(k, 70)$.

   (d) (3 points) How many elements of G have order 14?

   (e) (2 points) How many subgroups does G have?

3. Suppose $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 9 & 2 & 1 & 10 & 4 & 8 & 3 & 5 & 7 & 6 \end{pmatrix}$.
   (a) (3 points) Find a cycle decomposition of σ.

   (b) (2 points) Find $|\langle \sigma \rangle|$.

   (c) (3 points) Find a cycle decomposition of σ^{59}.

   (d) (2 points) Is σ odd or even? Justify your answer.