Homework 4

Thursday, April 26, 2018 8:24 AM

1. Prove that $\mathbb{Q}[x]/\langle x^2-2\rangle \simeq \mathbb{Q}[\sqrt{2}]$.

2. Prove that $\mathbb{Z}[i]/\langle 2+i\rangle \simeq \mathbb{Z}/5\mathbb{Z}$.

3. Suppose $m,n \in \mathbb{Z}^{\geq 2}$ and $\gcd(m,n)=1$. Prove that

$$\mathbb{Z}/mn\mathbb{Z} \simeq \mathbb{Z}/m\mathbb{Z} \times \mathbb{Z}/n\mathbb{Z}$$

(Hint. Let $c: \mathbb{Z} \to \mathbb{Z}/m\mathbb{Z} \times \mathbb{Z}/n\mathbb{Z}$, $c(k) := (k+m\mathbb{Z}, k+n\mathbb{Z})$

Show that c is a ring homomorphism and $\ker c = mn\mathbb{Z}$.

Use the 1^{st} isomorphism theorem and the pigeonhole principle to finish proof.)

4. Prove that $\mathbb{Z}[x]/n\mathbb{Z}[x] \simeq \mathbb{Z}_n[x]$.

5. Prove that $\mathbb{Q}[x]/\langle x^2-2x+6\rangle \simeq \{c_0+c_1A \mid c_0, c_1 \in \mathbb{Q}\}$

where $A = \begin{bmatrix} 0 & -6 \\ 1 & 2 \end{bmatrix}$.

(Hint. Consider $\phi_A: \mathbb{Q}[x] \to M_2(\mathbb{Q})$,

$\phi_A(\sum_{i=0}^{n} a_i x^i) = a_0 I + a_1 A + \cdots + a_n A^n$. Use without proof that

ϕ_A is a ring homomorphism.)