1. Suppose $R_1, ..., R_n$ are rings. Prove that $R_1, ..., R_n$ are unital if and only if $R_1 \times ... \times R_n$ is unital.

2. Suppose $R$ is a unital ring. An element $x$ of $R$ is called a unit if it has a multiplicative inverse; that means $\exists x' \in R$ such that $x \cdot x' = x' \cdot x = 1_R$.

Let $R^\times$ be the set of all the units of $R$.

(a) Prove that $R^\times$ is closed under multiplication.

(b) Prove that $(R^\times, \cdot)$ is a group.

(c) Suppose $R_i$'s are unital rings. Prove that

$$(R_1 \times ... \times R_n)^\times = R_1^\times \times ... \times R_n^\times.$$ 

(d) Find $(\mathbb{Z} \times \mathbb{Q})^\times$.

3. Show that $\mathbb{Z}[a+b\sqrt{3} \mid a, b \in \mathbb{Z}]$ is a subring of $\mathbb{R}$.

4. As in problem 3 one can show $F = \mathbb{Q}[a+b\sqrt{3} \mid a, b \in \mathbb{Q}]$ is a ring. Show that $F^\times$ is a field.
5. Suppose $A$ is a ring with unity $1$. Suppose there is $a_0 \in A$ such that $a_0^2 = 1$. Let $B = \{a_0 r a_0 | r \in A\}$. Prove that $B$ is a subring of $A$. 