Name: ______________________________

PID: ______________________________

Section: ______________________________

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

1. Write your Name, PID, and Section on the front page of your exam.
2. No calculators or other electronic devices are allowed during this exam.
3. Read each question carefully, and answer each question completely.
4. Write your solutions clearly in the exam sheet.
5. Show all of your work; no credit will be given for unsupported answers.

1. Compute the product in the given ring.
   (a) (4 points) $(12, 16)(16, 3)$ in $\mathbb{Z}_{24} \times \mathbb{Z}_{32}$.

   (b) (4 points) $\left(\begin{smallmatrix} 2 & 1 \\ 1 & 0 \end{smallmatrix}\right)\left(\begin{smallmatrix} 5 & 3 \\ 3 & 2 \end{smallmatrix}\right)$ in $M_2(\mathbb{Z}_6)$.

   (c) (2 points) $(2^{-1})(3)$ in $\mathbb{Z}_{11}$.

2. (10 points) Describe all the ring homomorphisms of $\mathbb{Z}\times\mathbb{Z}$ into $\mathbb{Z}_7$. Justify your answer.

3. Let $R = \mathbb{Z}_6 \times \mathbb{Z}_4 \times \mathbb{Z}_9$.

   (a) (5 points) Find the characteristic of R. Justify your answer.

   (b) (5 points) Find the number of units of R. Justify your answer.

4. (10 points) We are told that $\mathbb{Q}[\sqrt{2}] = \{a + b\sqrt{2} | \ a, b \in \mathbb{Q}\}$ is a subring of $\mathbb{R}$. Show that it is a field. (You do not have to prove that $\sqrt{2}$ is an irrational number.)

5. (10 points) Suppose R is a unital ring with no zero-divisors. Show that if $xy = 1$ for $x, y \in R$, then $yx = 1$. Justify your answer. (Hint: consider $(xy)x$)

Good Luck!