Math 109: The first exam.
Instructor: A. Salehi Golsefidy

Name: ....

PID: ....

10/18/2016

1. Write your Name and PID on the front of your exam sheet.
2. No calculators or other electronic devices are allowed during this exam.
3. Show all of your work; no credit will be given for unsupported answers.
4. Read each question carefully to avoid spending your time on something that you are not supposed to (re)prove.
5. Ask me or a TA when you are unsure if you are allowed to use certain fact or not.
6. Good luck!

Question	Points	Score
1	10	
2	17	
3	10	
4	3	
Total:	40	

1. (10 points) Which one of the following propositional forms is NOT equivalent to $P \Rightarrow (Q \vee R)$? Justify your answer.

    1. $(P \wedge (\neg Q)) \Rightarrow R$.
    2. $(P \wedge (\neg Q) \wedge (\neg R)) \Rightarrow \perp$, where $\perp$ means contradiction.
    3. $(P \Rightarrow Q) \wedge (P \Rightarrow R)$.
    4. $(\neg P) \vee Q \vee R$.
    5. $((\neg Q) \wedge (\neg R)) \Rightarrow (\neg P)$.

    (You have to only prove why your chosen propositional form is not equivalent to $P \Rightarrow (Q \vee R)$. You do NOT need to argue why the rest are equivalent.)

2. Determine if the following propositions are true or not. Briefly justify your answer.

   (a) (4 points) For any positive integers a, b, $10|ab$ implies that either $10|a$ or $10|b$.

   (b) (4 points) For integers a and b, $a|b$ implies $|a| \leq |b|$.

(c) (4 points) For any two real numbers x, y, we have $x^2 + y^2 \geq 2xy$.

(d) (5 points) There are integers m and n such that $21m - 14n = 2$.

3. (10 points) Let $a_0 = 0$, $a_1 = 1$, and $a_{n+1} = a_n + 6a_{n-1}$ for any positive integer n. Prove that for any positive integer n we have that $a_n = (3^n - (-2)^n)/5$.

4. (3 points) Suppose a_1, a_2, a_3 are integers. Prove that $(a_1 - a_2)(a_2 - a_3)(a_3 - a_1)$ is even. (Hint: $(a_1 - a_2) + (a_2 - a_3) + (a_3 - a_1) = 0$.)