Problem set 2

Monday, October 12, 2015 4:54 PM

1. (a) Prove that for any integer $n$, one and exactly one of the numbers $n$ and $n+1$ is even.

(b) Prove that, for any integer $n$,

$$n(n+1)$$

is even.

(Hint. For @ use the result that we proved in class:

For any integer $m$, $m$ is odd if and only if $m = 2k+1$ for some integer $k$.

For (b), use part @.)

2. Prove that $201x - 9y = 2$ has no integer solutions.

3. Prove that for any positive real numbers $x, y, z$

$$\sqrt{\frac{x^2 + y^2 + z^2}{3}} \geq \frac{x + y + z}{3}.$$  

(You are allowed to use whatever is proved in class.)

4. Determine if the following statements are true or not. Justify your answer.

(a) For any integers $m$ and $n$,

$$6 \mid mn \implies 6 \mid m \lor 6 \mid n.$$
(b) For any integers \( m \) and \( n \),
\[ 6 \mid m \land 6 \mid n \implies 6 \mid mn. \]

(c) For any integers \( m \) and \( n \),
\[ 3 \mid mn \implies 3 \mid m \lor 3 \mid n \]

(For part (c) you are allowed to use the following:
For any integer \( n \), \( 3 \mid n \iff \text{ for some integer } k, n = 3k \pm 1 \))

5. Let \( d \) be an integer more than 1, and \( a_1, a_2, b_1, \) and \( b_2 \)
are integers. Suppose \( d \mid a_1 - a_2 \) and \( d \mid b_1 - b_2 \).

Prove that \( d \mid (a_1 + b_1) - (a_2 + b_2) \)
and \( d \mid a_1 b_1 - a_2 b_2 \).

(Hint. For the second part use
\[ a_1 b_1 - a_2 b_2 = (a_1 - a_2) b_1 + a_2 (b_1 - b_2). \])