1. Let \( f: (0, 1] \rightarrow [-1, 1] \), for any \( k \in \mathbb{Z}^{>0} \),
\[
f(x) = (-1)^k \quad \text{if} \quad \frac{1}{2^{k+1}} < x \leq \frac{1}{2^k}.
\]
So its graph looks like:

![Graph of the function](image)

Prove that \( \lim_{x \to 0^+} f(x) \) does NOT exist.

(Hint: It is similar to the example that we did in class: \( \lim_{x \to 0} \sin\left(\frac{1}{x}\right) \) does NOT exist.)

2. (a) Prove or disprove: \( \forall x \in \mathbb{R}, \ (\forall \varepsilon > 0, |x| \leq \varepsilon \Rightarrow x = 0) \).

(b) Prove or disprove: \( \forall x \in \mathbb{R}, \forall \varepsilon > 0, \ (|x| \leq \varepsilon \Rightarrow x = 0) \).

3. Prove that \( A \times (B \cup C) = (A \times B) \cup (A \times C) \).

4. (a) Find all possible \( a \in \mathbb{R} \) such that
\[
\exists x \in \mathbb{R}, \ a^2 - 2x + a^2 = 0.
\]
(b) Find all possible $a \in \mathbb{R}$ such that
\[ \exists x \in \mathbb{R}, \ x^2 - 2x + a^2 = 0. \]

5. Prove that there are $2^n$ functions $f: \{1, 2, \ldots, n\} \to \{0, 1\}$.

(Hint. Use induction on $n$.)

6. For $A \subseteq X$, the characteristic function $1_A$ of $A$ is
\[ 1_A : X \to \{0, 1\}, \ 1_A(x) = \begin{cases} 1 & x \in A, \\ 0 & x \in X \setminus A. \end{cases} \]

(a) Prove that $1_{A \cap B} = 1_A \cdot 1_B$.

(b) Prove that $1_{A^c} + 1_A = 1_X$. 