Problem set 1
Tuesday, September 29, 2015 1:38 PM

1. Construct truth tables for the following propositional forms.

(i) $P \land (Q \lor R)$  
(ii) $\neg (P \to Q)$

(iii) $(P \land Q) \lor (P \land R)$  
(iv) $(-P) \land Q$

2. Use the first problem to deduce that

(i) $P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)$ (distributive law)

(ii) $\neg (P \to Q) \equiv P \land (\neg Q)$

3. Prove that for any positive real numbers $a$ and $b$ we have

$$\sqrt{ab} \geq \min \{a, b\}.$$  

4. Prove that for any real numbers $a$ and $b$ we have

$$|a+b| \leq |a| + |b|.$$  

(Hint: $x^2 \leq y^2 \iff |x| \leq |y|$ and $z \leq |z|$.)

5. Prove that for all integers $n$,

$$4(n^2+n+1) - 3n^2$$

is a perfect square.
6. We would like to color each circle in a way that two connected circles have different colors. What is the minimum number of needed colors? Justify your answer.

(i) 

(ii) 

(iii)