Problem set 5

Sunday, October 30, 2016 8:13 AM

1. Write down the negation of the following statements:

(a) $\forall \varepsilon > 0, \exists \delta > 0, |x-1| < \delta \Rightarrow |x^2-1| < \varepsilon$.

(b) $\forall \varepsilon > 0, \forall x \in \mathbb{R}, \exists n \in \mathbb{Z}, |x-n| < \varepsilon$

(c) Let α be an irrational number, i.e. $\alpha \in \mathbb{R} \setminus \mathbb{Q}$.

$$\forall \varepsilon > 0, \forall x \in \mathbb{R}, \exists m, n \in \mathbb{Z}, |x-m-n\alpha| < \varepsilon.$$

2. (a) Prove or disprove: $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, y^2 > 2016 + x$

(b) Prove or disprove: $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, y^3 > 2016 + x$

(c) Prove or disprove: $\forall \varepsilon > 0, \exists N \in \mathbb{Z}^{>0}, n \geq N \Rightarrow \frac{1000}{n} < \varepsilon$.

(For part (c), you are allowed to use the following:

$\forall x \in \mathbb{R}, \exists n \in \mathbb{Z}, x < n$.)

3. Prove that $\forall L_1, L_2 \in \mathbb{R}, ((\forall \varepsilon > 0, |L_1 - L_2| < \varepsilon) \Rightarrow L_1 = L_2)$.

4. (a) Use quantifiers to give a precise formulation of:
"the sequence x_n gets closer and closer to a."
(Hint A bit more precise statement would be:
"for large enough n, x_n is ε-close to a.")

(b) Suppose there are two sequeces x_n^+ and x_n^- which get closer and closer to a and at the same time $f(x_n^+)$ gets closer and closer to L_1 and $f(x_n^-)$ gets closer and closer to L_2 where $L_1 \neq L_2$. Prove $\lim_{x \to a} f(x)$ does NOT exist.