Math 20C.
Midterm Exam 1
April 25, 2007

Turn off and put away your cell phone.
You may use any type of handheld calculator; no other devices are allowed on this exam.
You may use one page of notes, but no books or other assistance on this exam.
Read each question carefully, answer each question completely, and show all of your work.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.

1. (4 points) Let \( \mathbf{a} = (-2, 1, 8) \) and let \( \mathbf{u} = (s, s^2, -1) \). Find the values of \( s \) for which \( \mathbf{a} \) and \( \mathbf{u} \) are orthogonal.
2. Let \( \mathbf{a} = (1, 1, 1) \) and \( \mathbf{b} = (-2, -2, 1) \).

(a) (2 points) Find two vectors that are orthogonal to both \( \mathbf{a} \) and \( \mathbf{b} \).

(b) (2 points) Find the sine of the angle between \( \mathbf{a} \) and \( \mathbf{b} \).
3. (6 points) Find an equation for the plane containing the lines $\mathbf{\lambda}(t) = \langle 1, 3, 5 \rangle + t\langle 1, 4, 7 \rangle$
and $\mathbf{\mu}(t) = \langle 1, 3, 5 \rangle + t\langle 2, -1, 4 \rangle$. 
4. A particle’s position function is $\mathbf{r}(t) = \langle 2\cos(t), 2\sin(t), 2t \rangle$ for $0 \leq t \leq 2\pi$.

(a) (2 points) Find the particle’s velocity $\mathbf{v}(t)$ and speed $|\mathbf{v}(t)|$ as a function of time $t$.

(b) (2 points) Find the particle’s acceleration $\mathbf{a}(t)$ as a function of time $t$.

(c) (3 points) Find the angle between the particle’s position $\mathbf{r}(t)$ and acceleration $\mathbf{a}(t)$ as a function of time $t$.

(d) (3 points) Determine how far the particle traveled during the time interval $0 \leq t \leq 2\pi$. 

5. (a) (3 points) Evaluate \( \lim_{(x,y) \to (0,0)} \frac{2(x^2 + y^2)}{\sqrt{x^2 + y^2 + 1} - 1}. \)

(b) (3 points) Explain why \( \lim_{(x,y) \to (0,0)} \frac{x^2}{2x^2 + y^2} \) does not exist.