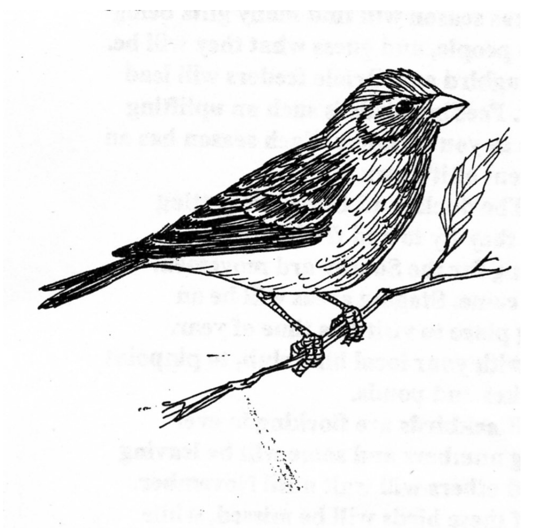
Name:______________________________
Section:______________________________
TA:______________________________

Math 20C
Exam 1 A
October 21, 2011



Turn off and put away your cell phone.
You may use a calculator and one sheet of notes on the exam.
Read each question carefully, and answer each question completely.
Show all of your work; no credit will be given for unsupported answers.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.

#	Points	Score
1	9	
2	9	
3	12	
4	10	
Σ	40	

1. (9) Suppose

$$\mathbf{u} = \langle 4, 1, -2 \rangle, \quad \mathbf{v} = \langle -3, 6, 1 \rangle, \quad \mathbf{w} = \langle -2, -1, 3 \rangle$$

Compute

(a) $\mathbf{u} \cdot \mathbf{w} = (4)(-2) + (1)(-1) + (-2)(3)$

$= -8 - 1 - 6$

$= -15$

(b) $\mathbf{w} \times \mathbf{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & -1 & 3 \\ -3 & 6 & 1 \end{vmatrix} = \left\langle \begin{vmatrix} -1 & 3 \\ 6 & 1 \end{vmatrix}, -\begin{vmatrix} -2 & 3 \\ -3 & 1 \end{vmatrix}, \begin{vmatrix} -2 & -1 \\ -3 & 6 \end{vmatrix} \right\rangle$

$= \langle (-1)(1) - (3)(6), -[(-2)(1) - (3)(-3)], (-2)(6) - (-1)(-3) \rangle$

$= \langle -1 - 18, -[-2 + 9], -12 - 3 \rangle = \langle -19, -7, -15 \rangle$

(c) $||\mathbf{w}|| = \sqrt{(-2)^2 + (-1)^2 + (3)^2}$

$= \sqrt{4 + 1 + 9}$

$= \sqrt{14}$.

2. (9)

Consider the path

$$\mathbf{r}(t) = \left\langle \frac{1}{1+t}, \cos \pi t, 3t^2 \right\rangle$$

Find a parametrization of the line that is tangent to $\mathbf{r}(t)$ when $t = 1$.

A parametrization of the line that is tangent to $r(t)$ when $t = t_0$ is $L(t) = r(t_0) + t\, r'(t_0)$. So

$$r'(t) = \left\langle \frac{-1}{(1+t)^2}, -\pi \sin \pi t, 6t \right\rangle$$

Therefore

$$r'(1) = \left\langle \frac{-1}{4}, 0, 6 \right\rangle \text{ and}$$

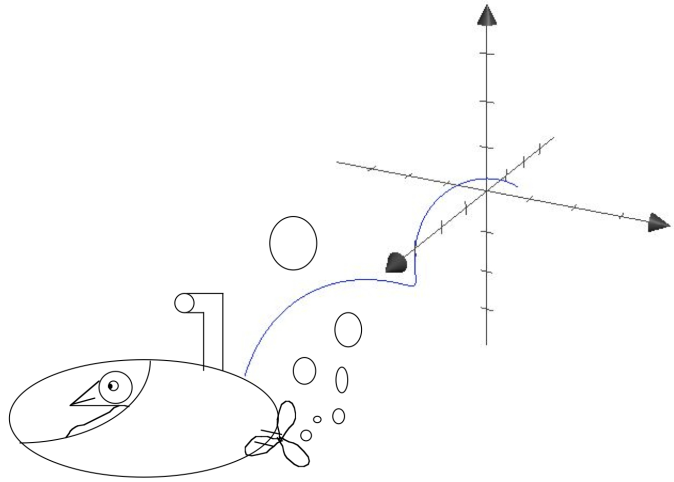
$$r(1) = \left\langle \frac{1}{2}, -1, 3 \right\rangle, \text{ which implies}$$

$$L(t) = \left\langle \frac{1}{2}, -1, 3 \right\rangle + t \left\langle \frac{-1}{4}, 0, 6 \right\rangle$$

$$= \left\langle \frac{1}{2} - \frac{t}{4}, -1, 3+6t \right\rangle$$

is a parametrization of the tangent line of $r(t)$ at $r(1)$.

3. (12) Suppose a bird is traveling in a submarine along the path $\mathbf{r}(t) = \langle 3t - 1, \cos 2t, \sin 2t \rangle$.



(a) Compute the length of its path from $t = 0$ to $t = 5$.

$$\int_0^5 \|r'(t)\|\,dt = \int_0^5 \|\langle 3, -2\sin 2t, 2\cos 2t\rangle\|\,dt$$
$$= \int_0^5 \sqrt{9 + 4\sin^2 2t + 4\cos^2 2t}\,dt = \int_0^5 \sqrt{13}\,dt$$
$$= 5\sqrt{13}.$$

(b) Compute the velocity at time t.

$$v(t) = r'(t) = \langle 3, -2\sin 2t, 2\cos 2t \rangle.$$

(c) Compute the acceleration at time t.

$$a(t) = v'(t) = \langle 0, -4\cos 2t, -4\sin 2t \rangle$$

(d) At time $t = \pi/2$, is the submarine speeding up, slowing down or traveling at a constant speed? Explain your answer.

Solution 1 Speed of the submarine at time $t = \|v(t)\| = \sqrt{9 + 4\sin^2 2t + 4\cos^2 2t} = \sqrt{13}$. So it has a constant speed. Therefore it always (in parti. at time $t = \pi/2$) travels at a constant speed.

Solution 2 To see if a moving particle is speeding up or not, we should look at $\vec{a} \cdot \vec{v}$:

$\vec{a} \cdot \vec{v} > 0 \Rightarrow$ speeding up
$\vec{a} \cdot \vec{v} < 0 \Rightarrow$ slowing down
$\vec{a} \cdot \vec{v} = 0 \Rightarrow$ constant speed

In our case, $\vec{a} \cdot \vec{v} = 0$.

4. (10)

(a) Determine the equation of the plane that passes through the points

$$\underset{P}{(2,-1,4)}, \quad \underset{Q}{(1,-1,3)}, \quad \underset{R}{(1,3,-2)}.$$

$\vec{n} = \vec{PQ} \times \vec{PR}$ is a normal vector

$\vec{PQ} = \langle 1-2, -1+1, 3-4 \rangle = \langle -1, 0, -1 \rangle$

$\vec{PR} = \langle 1-2, 3+1, -2-4 \rangle = \langle -1, 4, -6 \rangle$

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 0 & -1 \\ -1 & 4 & -6 \end{vmatrix} = \langle 4, -5, -4 \rangle.$$

Since the considered plane passes through P:

$4x - 5y - 4z = (4)(2) - (5)(-1) - (4)(4) = 8 + 5 - 16 = -3.$

(b) Find the point of intersection of the plane

$$2x - 3y + z = 5$$

and the line

$$\mathbf{r}(t) = \langle 3t+2, 1, -7t \rangle.$$

The given line is

$x = 3t+2$, $y = 1$ and $z = -7t$

So intersection happens when

$2(3t+2) - 3(1) + (-7t) = 5$, which implies

$6t + 4 - 3 - 7t = 5$

$\Rightarrow -t + 1 = 5 \Rightarrow t = -4$

$\Rightarrow$ the point of intersection is

$\langle (3)(-4)+2, 1, (-7)(-4) \rangle = \langle -10, 1, 28 \rangle$