Name: 

PID: 

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1. Let $A = (1, 0, 3)$ and $B = (-3, 2, 1)$.
   (a) (3 points) Normalize $\overrightarrow{AB}$.
   (b) (3 points) Find the midpoint $M$ of the segment $AB$.
   (c) (4 points) Find equation of the plane passing through $M$ and perpendicular to $\overrightarrow{AB}$.

2. (5 points) Evaluate the limit or determine that it does not exist.
   \[
   \lim_{(x,y) \to (0,0)} \frac{xy}{x^2 + y^2}.
   \]

3. Answer the following questions with short justifications:
   (a) (3 points) Find the length of $\frac{-3}{\|v\|} \mathbf{v}$.
   (b) (4 points) Suppose $\mathbf{v} \times \mathbf{w} = (1, -2, 1)$ and $\mathbf{u} = (1, 1, -1)$. Find the volume of the parallelepiped spanned by $\mathbf{v}, \mathbf{w}$ and $\mathbf{u}$.
   (c) (4 points) Suppose $\|\mathbf{v} \times \mathbf{w}\| = 3$. Find the area of the parallelogram spanned by $2\mathbf{v} + 3\mathbf{w}$ and $\mathbf{v} + \mathbf{w}$.
   (d) (4 points) Suppose $\|\mathbf{v}\| = 2$, $\|\text{proj}_\mathbf{v} \mathbf{w}\| = 5$, and the angle between $\mathbf{v}$ and $\mathbf{w}$ is obtuse. Find $\mathbf{v} \cdot \mathbf{w}$.
   (e) (3 points) Find a normal vector of a plane which is parallel to the line $L(t) = t(1, 2, 3) + (1, 0, 1)$ and perpendicular to the plane $x - y + z = 1$.
   (f) (3 points) Find a vector parallel to the line of intersection of the planes $x + y + z = 1$ and $-x + y - z = 0$. 
4. (4 points) Match the following functions with the contour diagrams (a)-(d).

1. \( f_1(x, y) = x^3 - y \)
2. \( f_2(x, y) = xy \)
3. \( f_3(x, y) = x^2 - y^2 \)
4. \( f_4(x, y) = y - \ln x \)

Good Luck!
1(a) \[ \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (-3, 2, 1) - (1, 0, 3) = (-4, 2, -2) \]

\[ \|\overrightarrow{AB}\| = \sqrt{(-4)^2 + 2^2 + (-2)^2} = \sqrt{16 + 4 + 4} = \sqrt{24}. \]

Normalizing \[ \overrightarrow{AB} : \frac{\overrightarrow{AB}}{\|\overrightarrow{AB}\|} = \left( \frac{-4}{\sqrt{24}}, \frac{2}{\sqrt{24}}, \frac{-2}{\sqrt{24}} \right) \]

\[ = \left( -\frac{\sqrt{24}}{6}, \frac{\sqrt{24}}{12}, -\frac{\sqrt{24}}{12} \right). \]

1(b) \[ \overrightarrow{OM} = \frac{1}{2} \overrightarrow{OA} + \frac{1}{2} \overrightarrow{OB} = \left( \frac{-3+1}{2}, \frac{2+0}{2}, \frac{1+3}{2} \right) \]

\[ = (-1, 1, 2). \] So \( M = (-1, 1, 2). \)

1(c) \[ -4x + 2y - 2z = (4)(-1) + 2(1) - 2(2) \]

\[ = 4 - 2 - 4 = 2. \]

So \[ -2x + y - z = 1. \]

2. Let's approach to \((0,0)\) along the line \( y = kx \).

\[ \lim_{x \to 0} \frac{x(kx)}{x^2 + (kx)^2} = \lim_{x \to 0} \frac{kx^2}{(1+k^2)x^2} = \frac{k}{1+k^2} \]

Since this limit depends on \( k \), \( \lim_{(x,y) \to (0,0)} \frac{xy}{x^2 + y^2} \) does NOT exist.

3(a) \[ \frac{\vec{V}}{\|\vec{V}\|} \] is a unit vector. So length of \( -3 \frac{\vec{V}}{\|\vec{V}\|} \) is \( |-3| = 3 \).
3(b) Volume of the parallelepiped spanned by \( \vec{v}, \vec{w}, \) and \( \vec{u} \)

\[
= |(\vec{v} \times \vec{w}) \cdot \vec{u}| = |(1, -2, 1) \cdot (1, 1, -1)|
\]

\[
= |1 - 2 - 1| = 2.
\]


3(c) Area of the parallelogram spanned by \( 2 \vec{v} + 3 \vec{w} \) and \( \vec{v} + \vec{w} \)

\[
\vec{v} + \vec{w} = \| (2 \vec{v} + 3 \vec{w}) \times (\vec{v} + \vec{w}) \|
\]

\[
= \| 2 \vec{v} \times \vec{v} + 2 \vec{v} \times \vec{w} + 3 \vec{w} \times \vec{v} + 3 \vec{w} \times \vec{w} \|
\]

\[
= \| 2 \vec{v} \times \vec{w} - 3 \vec{v} \times \vec{w} \| = \| -\vec{v} \times \vec{w} \| = 3.
\]

Lecture 6, page 4.

3(d) \( |\vec{v} \cdot \vec{w}| = \| \text{Proj}_\vec{v} \vec{w} \| \| \vec{v} \| = (5)(2) = 10 \)

Since the angle between \( \vec{v} \) and \( \vec{w} \) is obtuse, \( \vec{v} \cdot \vec{w} < 0 \).

So \( \vec{v} \cdot \vec{w} = -10 \).

Lecture 5, page 1, 2.

3(e) Both normal vector of \( x - y + z = 1 \) and a vector parallel to the line \( \vec{l}(t) = t(1, 2, 3) + (1, 0, 1) \) are parallel to the plane that we are looking for. So \( \vec{v} = (1, -1, 1) \) and \( \vec{w} = (1, 2, 3) \) are parallel to this.
plane. Hence 
\[ \vec{V} \times \vec{W} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = (-3-2, 1-3, 2+1) \]
\[ = (-5, -2, 3) \]
is a normal vector of this plane.

Section 1.3, Problem 35
(This was a bit tricky, and I treated this as a bonus problem.)

3(f) The line of intersection would be perpendicular to normal vectors of the planes \( x+y+z=1 \) and \(-x+y-z=0 \)

So it is parallel to their cross product:
\[ (1,1,1) \times (-1,1,-1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{vmatrix} = (-2-1, -1+1, 1+1) \]
\[ = (-3, 0, 2) \]

Most parts of problem 3 are similar to problem 4 in the fifth practice exam.

4. 1 \( x^2-y=0, 1, -1 \Rightarrow y = x^3 \text{ or } x^3 + 1 \Rightarrow b \)

2 \( xy = 1 \text{ is a level curve} \Rightarrow a \)

3 \( x^2-y^2=0, 1, 2 \Rightarrow \text{hyperbolas and crossed lines} \Rightarrow a \)

4 \( y - \ln x = 0 \Rightarrow y = \ln x \Rightarrow c \)

Lecture 9, page 4.