Name: ________________________________________________

PID: ________________________________________________

Section: ____________________________________________

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1. A particle’s position function is \( \mathbf{r}(t) = (\ln t, t^2, 2t + 1) \).
   (a) (3 points) Find the particle’s velocity \( \mathbf{v}(t) \) and its acceleration \( \mathbf{a}(t) \).
   (b) (2 points) Find the particle’s speed \( ||\mathbf{v}(t)|| \) as a function of \( t \). (Simplify your answer)
   (c) (3 points) Find the total distance traveled by the particle during the time interval \( 1 \leq t \leq 2 \).

2. Evaluate the limit or determine that it does not exist.
   (a) (5 points) \( \lim_{(x,y) \to (1,0)}(x^2 - 1) \cos \left( \frac{1}{(x-1)^2+y^2} \right) \).
   (b) (5 points) \( \lim_{(x,y) \to (0,0)} \frac{xy^2}{x+y} \).

3. Let \( f(x,y) = \cos(x^2+y) \) and \( P_0 = (1, \pi/2 - 1, 0) \).
   (a) (3 points) Find \( \nabla f(x,y) \).
   (b) (5 points) Find the equation of the tangent plane of \( z = f(x,y) \) at \( P_0 \).
   (c) (2 points) Find the maximum rate of increase of \( f \) at \( (1, \pi/2 - 1) \).

4. Answer the following questions with short justifications:
   (a) (2 points) Suppose \( \nabla f(1,2) = (-1,3) \) for some function \( f \). Is \( f \) increasing or decreasing in the direction of \( \mathbf{v} = (2,1) \).
   (b) (2 points) Find a normal vector of the tangent plane of the hyperboloid \( \frac{x^2}{4} + y^2 - \frac{z^2}{2} = 1 \) at \( (2,1,3) \).
   (c) (3 points) Find \( \frac{\partial z}{\partial y} \) where \( z = f(x,y) \) satisfies \( e^{xy} + \sin(xz) + y = 0 \).
      (Your answer can be in terms of \( x, y, \) and \( z \).)
   (d) (3 points) Let \( x = s + t \) and \( y = s - t \). Show that for any differentiable function \( f(x,y) \) we have \( f_x^2 - f_y^2 = f_x f_t \).
   (e) (2 points) We are told that the velocity of a particle is \( (-1,-2) \) and its acceleration is \( (-3,1) \). Is the particle slowing down or speeding up?

Good Luck!