INTRODUCTION

“Most of calculus is easy, once you figure it out.”

The above statement, which is the basic premise of this text, might come as a surprise to you and requires some explanation.

Firstly, the statement does not say that calculus is easy. It is not. Calculus is a complex and abstract subject that has been developed over the past 2500 years and is used to study a wide variety of topics, including the effects of gravity and air resistance on the velocity and acceleration of vehicles and projectiles, the motion of planets and satellites, areas and volumes, probability, chemical reaction rates, air currents and ocean waves, electricity and magnetism, radioactive decay, the effects of supply and demand on market prices, and the growth of organisms and populations.

Secondly, the statement speaks of “figuring out calculus” and not of “learning calculus” to make an important point. When someone tells you how to change a tire on your car or how to load film in your new camera, you do not learn the exact words used in conveying the information or every detail of the instructions. You accomplish your goal by experimenting with the car or camera, and your memory later of how to perform the actions is based more on your experience than on what you were told.

The process of mastering mathematics is similar. You come to understand a new mathematical idea by working with it and by figuring out what it means to you based on your experience and on the ways you formulate and remember abstract ideas. You need to do much more than listen to your instructor and study the explanations given in your textbook.

To do your best, you need to approach every new mathematical topic by asking yourself, “What is this really about?” and “How can I explain this so it makes more sense to me?” If you approach the topic as a mystery to be unraveled, rather than just as information and procedures to be learned or memorized, you will bring out the best of your intellectual abilities. Then, as you work with the topic and the new concepts and techniques it entails, it will be come clearer and more meaningful to you until it is, in fact, easy to understand. At that point your sense of the topic will be based on your experience with it and may be as dissimilar to the explanations in your textbook or lectures as your understanding of how to change a tire or load your camera might differ from the exact words used to tell you how to do it.

This book is designed to help you to figure out calculus—with the assistance of your instructors—so it can be a useful tool in your later work and studies.

Many calculus exercises require fairly complex lines of reasoning and calculations. To solve them effectively, you need to write down every step completely and accurately and proceed carefully from step to step. With your calculations and conclusions clearly recorded on your paper, you will be able to concentrate on the details. Think of the diagram on the right at every stage as you solve a problem.†

Suppose, for instance, that an exercise asks for the radius of a circle of area $5\pi$ square feet. After reading the exercises, you might think, “I need to use the formula $A = \pi r^2$ for the area of a circle of radius $r$.” You write down the formula and look at it closely, leading you to think, “I need to set $A$ equal to $5\pi$,” and you write “$5\pi = \pi r^2$.”

†Diagram suggested by Rick LeBorne
After reading this equation you might say to yourself, “I need to divide both sides of the equation by \( \pi \),” and you write “\( 5 = r^2 \).” You look at this and think, “When I take the square root of both sides, I need to use \( \pm \),” and you write “\( r = \pm \sqrt{5} \).” Reading this, you probably realize that the negative value is irrelevant, and you write “\( r = \sqrt{5} \).” After rereading the exercises to be sure you answered it correctly, you might notice that the area was given in square feet, so that the radius is \( r = \sqrt{5} \) feet.

Review the precalculus Chapter 0, even if it is not assigned, Study Sections 0.1 through 0.3 at the beginning of the course and Sections 0.4 through 0.6 before you study Chapter 3.

Buy and use a graphing calculator to generate graphs and check answers as you do your homework, even if calculators are not required in your course. The text web page contains directions for using several popular models. Purchase a lined plastic ruler for drawing graphs and diagrams.

Read (and reread) the definitions, theorems, and proofs. The theory of calculus is generally not easy to follow when you begin studying a topic. Later, however, after you have worked with the topic for a few weeks, you will find that the formal statements in the theory do, in fact, provide a useful summary of ideas, results, and procedures.

You will see that, in addition to the solved examples that are found in most mathematics textbooks, many discussions in this book include questions for you to answer as you read. Some questions ask you to interpret results, some have you generate graphs on your calculator or computer so you will see geometric meanings of the solutions, and others have you check results of examples. The goal of the questions is to keep you actively involved in the logical development and to lead you think more about what is being presented.

Spend some time on each of the questions and think carefully about your response. When you are satisfied with your conclusion, compare it with the response given at the end of the section. If you have trouble with a question, do not read the response in the book right away. Continue reading and return to the question later, when you can approach it with fresh ideas. Ask other students or your instructor for clarification if necessary.

Work through the interactive examples that are listed before the exercise set in each section. The interactive solutions are on a CD that accompanies your text. It can be run with any computer browser without having to be connected to the internet. The interactive solutions provide dialogs much like instructors might use in office hours. You are asked what you need to do at each step in a solution and then asked to carry out the necessary calculations. These examples will help you if you have trouble with your homework, or you can use them after you have finished the homework to increase your understanding and command of the relevant problem-solving techniques as you prepare for your examinations.

The regular exercises are divided into “conceptual”, “basic”, and “exploratory” sets. The conceptual exercises explore the underlying algebraic and geometric ideas in the section. The basic exercises cover topics that are included in most calculus courses. The exploratory exercises either deal with other topics or are more challenging or involve more complex applications than the basic exercises. The conceptual exercises will help you understand the material. Solve all of the basic and exploratory exercises that are assigned, as well as other exercises of the same types until you have mastered the associated techniques.

The answer section at the back of the book includes outlines of solutions of selected exercises. These help you work the exercises by providing the results of intermediate steps in their solutions.

Arrange to study with other students in your class. You can solve harder exercises and write better solutions by working together, and discussing ideas with others is a natural way to improve your own understanding.