(9/30/08)

Math 10A. Lecture Examples.

Section 2.1. How do we measure speed?[†]

Example 1 **Imagine that a pilot is flying a small airplane toward the west from an airport and that the plane is $\mathbf{s(t) = t^3 + 30t + 100}$ miles from the airport t hours after noon (Figure 1). (a) What is the plane's average velocity from $\mathbf{t = 1}$ to $\mathbf{t = 5}$? (b) Draw the secant line whose slope is this average value.**

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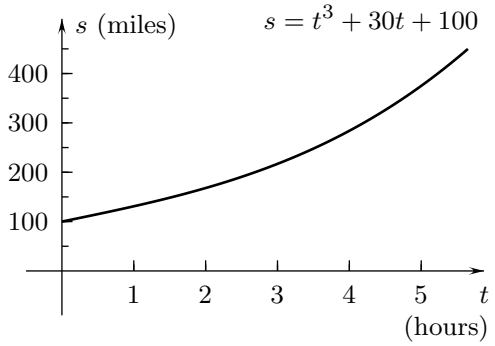


FIGURE 1

Answer: (a) [Average velocity for $1 \le t \le 5$] = 61 miles per hour **(b)** Figure A1

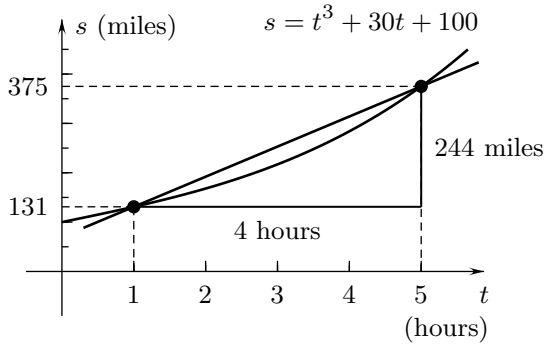


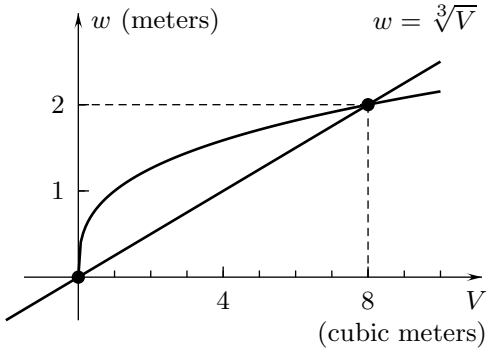
Figure A1

[†]Lecture notes to accompany Section 2.1 of *Calculus* by Hughes-Hallett et al.

Example 2 **What is the average rate of change of the width $\mathbf{w(V) = \sqrt[3]{V}}$ (meters) of a cube with respect to its volume $\mathbf{V}$ (cubic meters) from $\mathbf{V = 0}$ to $\mathbf{V = 8}$?**

Answer: [Average rate of change] $= \frac{1}{4}$ meters per cubic meter • Figure A2

Figure A2



Example 3 **Bats are warm-blooded mammals whose body temperatures keep fairly constant when they are awake and active. When a bat is asleep in a cold place, however, it goes into a sort of hybernation and its metabolism (rate of energy expenditure) drops. The next table gives the metabolism $\mathbf{r}$ of a sleeping brown bat as a function of the air temperature $\mathbf{T}$ around it.[(1)] (a) What is the average rate of change with respect to temperature of the bat's metabolism for $\mathbf{20 \le T \le 30}$? (b) What is the average rate of change with respect to temperature of the bat's metabolism for $\mathbf{0.5 \le T \le 2.0}$? Interpret its sign.**

T = Temperature (°C)	0.5	2.0	10.0	20.0	30.0	37.0	41.5
r = Metabolism (Calories/hour)	5.4	1.4	3.4	19.0	96.0	134.0	200.0

Answer: (a) [Average rate of change for $20 \le T \le 30$] = 7.7 Calories per hour per degree
(b) [Average rate of change for $0.5 \le T \le 2.0$] $= -\frac{8}{3}$ Calories per hour per degree • This average rate of change is negative because the bat's metabolism increases as the temperature drops from 2°C toward freezing (0°C).

[(1)] Data adapted from *Listening in the Dark, The Acoustic Orientation of Bats and Men*, Donald R. Griffin, Cornell University Press, Ithaca, N.Y., 1986, p. 40.

Example 4 **Figure 2 shows the hours of sunshine $\mathbf{H = H(t)}$ in Ft. Vermillion, Alberta, Canada as a function of the time of year $\mathbf{t}$ measured in months with $\mathbf{t = 0}$ at the beginning of the year.[2] What is the approximate average rate of change of the hours of sunshine with respect to time from the beginning of April at $\mathbf{t = 3}$ to the beginning of July at $\mathbf{t = 6}$?**

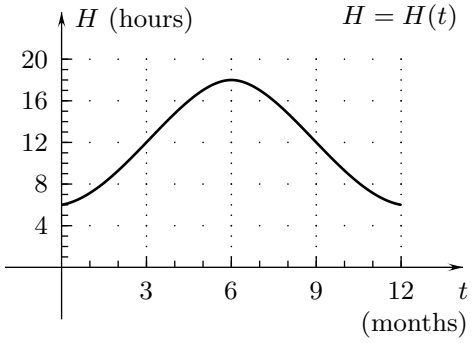


FIGURE 2

Answer: Figure A4 ● [Average rate of change] $\approx$ 2 hours per month

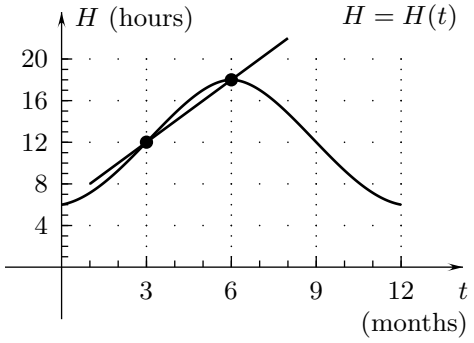


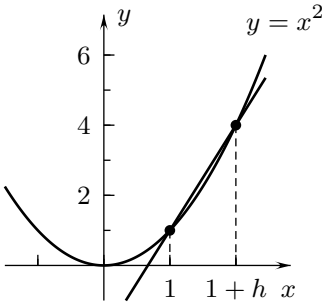
Figure A4

[2] Data adapted from *Introduction to Physical Geography* by A. Strahler, Second Edition, New York NY: John Wiley & Sons, Inc., 1970, p. 172.

Example 5 **Predict the derivative $\mathbf{f'(1)}$ of $\mathbf{f(x) = x^2}$ by calculating its average rate of change $\dfrac{\mathbf{(1+h)^2 - 1}}{\mathbf{h}}$ for $\mathbf{h = 1, h = 0.5, h = 0.25, h = 0.1, h = 0.001,}$ and $\mathbf{h = 0.00001.}$**

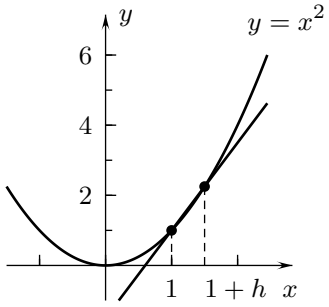
Answer: The average rates of change are in the table below and the corresponding secant lines are in Figures A5a through A5f. • Prediction: $f'(1) = 2$

h	1	0.5	0.25	0.1	0.001	0.00001
$\dfrac{(1+h)^2 - 1}{h} =$	3	2.5	2.25	2.1	2.001	2.00001



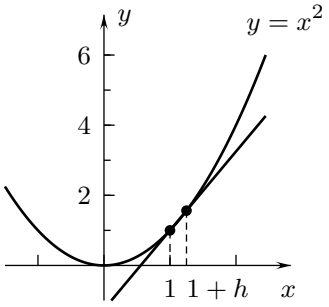
$h = 1$

Figure A5a



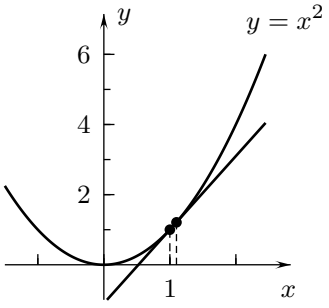
$h = 0.5$

Figure A5b



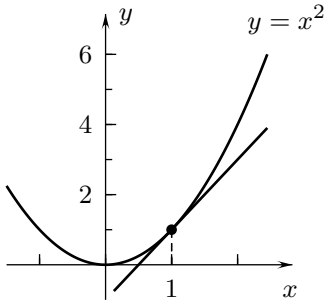
$h = 0.25$

Figure A5c



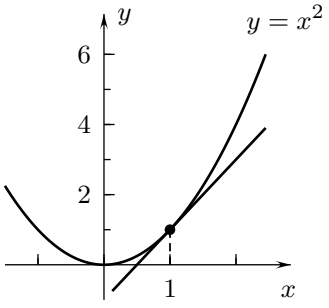
$h = 0.1$

Figure A5d



$h = 0.001$

Figure A5e



$h = 0.00001$

Figure A5f

Example 6 **Use the definition to find the derivative of $\mathbf{f(x) = x^2}$ at $\mathbf{x = 1}$.**

Answer: $f'(1) = 2$

Interactive Examples

Work the following Interactive Examples on Shenk's web page, http//www.math.ucsd.edu/˜ashenk/:[‡]

Section 2.2: Examples 1 through 4

Section 2.3: Example 5 ($\Delta x = h$)

[‡]The chapter and section numbers on Shenk's web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.