(9/30/08)

Math 10A. Lecture Examples.

Sections 2.2 and 2.3. The derivative at a point and as a function[†]

Example 1 **The function $\mathbf{A = A(t)}$ whose graph is shown in Figure 1 gives the percentage of alcohol in a person's blood $\mathbf{t}$ hours after he has consumed three fluid ounces of alcohol.[(1)] The tangent line at $\mathbf{t = 1}$ in Figure 2 passes through the points $\mathbf{P = (1,\ 0.18)}$ and $\mathbf{Q = (2,\ 0.28)}$. What is the (instantaneous) rate of change of the person's blood alcohol level with respect to time one hour after consuming the alcohol?**

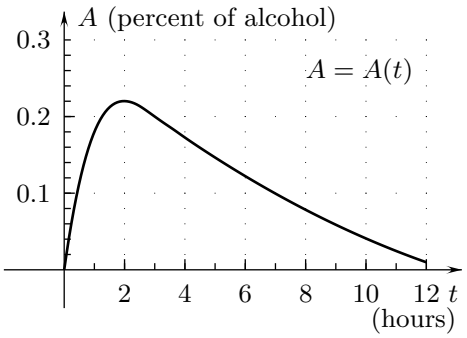


FIGURE 1

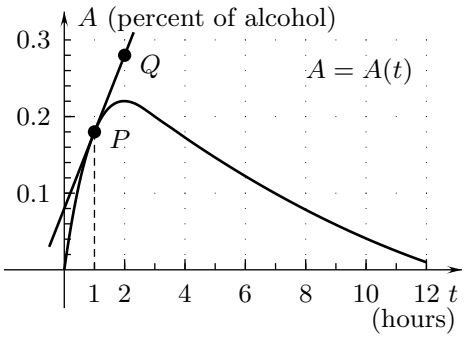


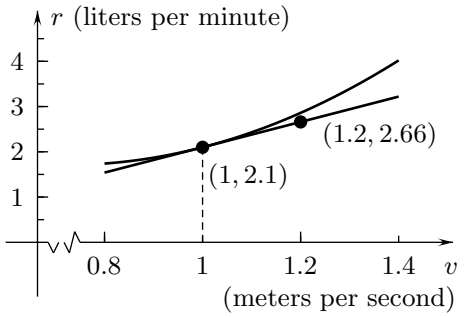
FIGURE 2

Answer: $A'(1) = 0.1$ percent per hour

Example 2 **Figure 3 shows the graph of the rate of oxygen consumption $\mathbf{r = r(v)}$ (liters per minute) of a swimmer as a function of his velocity $\mathbf{v}$ (meters per second) and the tangent line to the graph at $\mathbf{v = 1}$.[(2)] (a) Why is $\mathbf{r(1.4)}$ greater than $\mathbf{r(1)}$? (b) What is the rate of change of his oxygen consumption with respect to velocity at $\mathbf{v = 1}$? (Give the units.)**

$r = r(v)$

FIGURE 3



Answer: (a) $r(1.4)$ is greater than $r(1)$ because the swimmer uses more oxygen when he swims faster.

(b) $r'(1)$ = [Slope of the tangent line] $= \dfrac{2.66 - 2.1}{1.2 - 1} = 2.8$ liters per minute per meters per second.

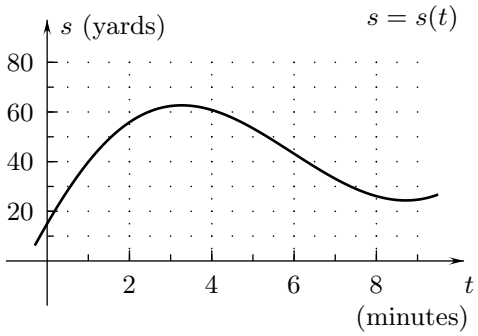
[†]Lecture notes to accompany Sections 2.2 and 2.3 of *Calculus* by Hughes-Hallett et al.

[(1)]Data adapted from *Encyclopædia Britannica*, Vol. 1, Chicago: Encyclopædia Britannica, Inc., 1965, p. 548.

[(2)]Data adapted from *The Human Machine* by R. Alexander, New York, NY: Columbia University Press, 1992, p. 117.

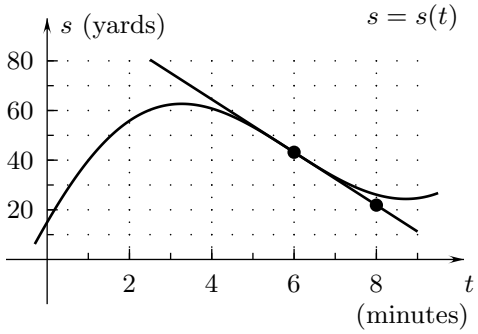
Example 3 **Figure 4 shows the graph of an object's position $s = s(t)$ on an s-axis as a function of the time t. What is the object's approximate velocity in the positive s-direction at $t = 6$?**

FIGURE 4



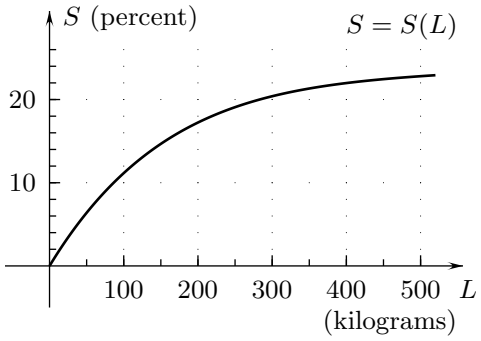
Answer: One answer: Figure A3 ● [Velocity at $t = 6$] $= s'(6) \approx -10.5$ yards per minute

Figure A3



Example 4 **Vertebrae in the human spine are separated by fibrous elastic disks that compress when the load on them increases. The load on the disk is called the *stress*, and the percent of compression is called the *strain*. The function $\mathbf{S = S(L)}$ in Figure 5 gives the strain $\mathbf{S}$ of lower back (lumbar) vertebral disks as a function of the stress $\mathbf{L}$ on them.[3] Sketch the graph of the derivative $\mathbf{r = S'(L)}$ of the stress function $\mathbf{S = S(L)}$.**

FIGURE 5



Answer: Figure A4a • $S'(50) \approx 0.11$ percent per kilogram • $S'(400) \approx 0.02$ percent per kilogram • Figure A4b

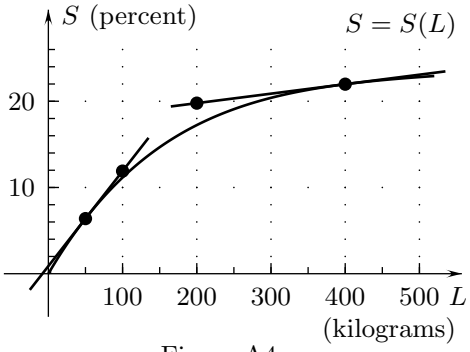


Figure A4a

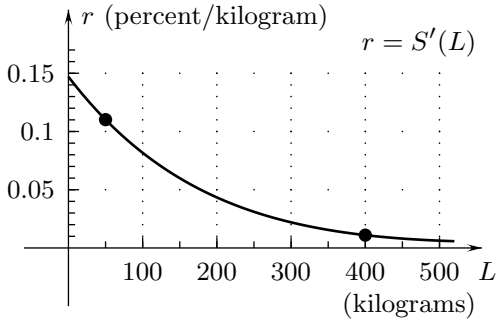


Figure A4b

[3] Data adapted from *Physics with Illustrative Examples from Medicine and Biology*, Vol. 1, G. Benedeck and F. Villars, Reading, Massachusetts: Addison-Wesley, 1973, p.343.

Example 5 **The next table lists the wind speed at three-hour intervals one day in Dodge City, Kansas.[4] The time t is measured in hours with $t = 0$ at midnight. Use this data to estimate the rate of change $W'(t)$ of the wind's velocity with respect to time at 9:00 AM ($t = 9$).**

Wind velocity $\mathbf{W(t)}$ ***(miles per hour) in Dodge City, Kansas***

t	0	3	6	9	12	15	18	21	24
$W(t)$	12.6	12.4	12.6	15.3	16.2	15.7	15.0	11.3	12.9

Answer: One approach: Use the secant line in Figure A5a (a left difference quotient). ●
$W'(9) \approx 0.3$ miles per hour per hour ●
Another approach: Use the secant line in Figure A5b (a right difference quotient). ●
$W'(9) \approx 0.9$ miles per hour per hour ●
A third approach: Use the secant line in Figure A5c (a centered difference quotient). ●
$W'(9) \approx 0.6$ miles per hour

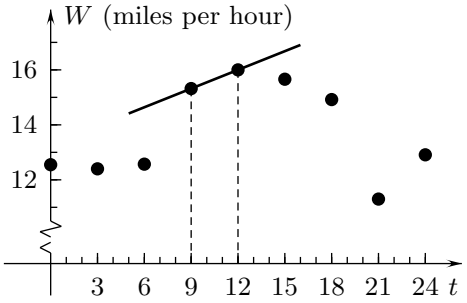


Figure A5a

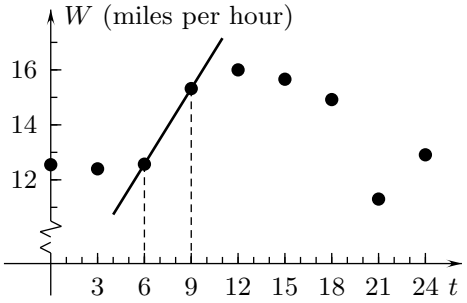


Figure A5b

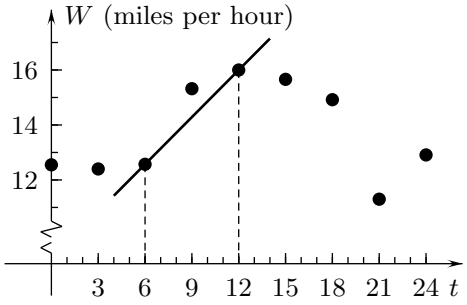


Figure A5c

Interactive Examples

Work the following Interactive Examples on Shenk's web page, http//www.math.ucsd.edu/˜ashenk/:‡

Section 2.5: Examples 1 through 5

[4] Data from *Wind Energy Systems*, G. L. Johnson, Englewood Cliffs, N. J: Prentice Hall International, Inc., 1985, p. 47.

‡ The chapter and section numbers on Shenk's web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.