(9/30/08)

Math 10A. Lecture Examples.

Sections 4.3 and 4.5. Optimization and modeling[†]

Example 1 **Find the maximum and minimum values of $\mathbf{h(x) = \dfrac{x}{x^2 + 4}}$ for $\mathbf{x \geq 0}$, if they exist.**

Answer: [Maximum] = $\frac{1}{4}$ • [Minimum] = 0 • (The graph is in Figure A1.)

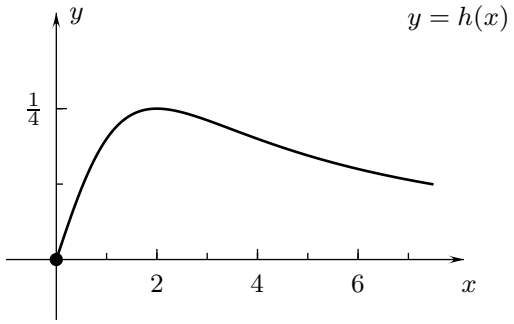
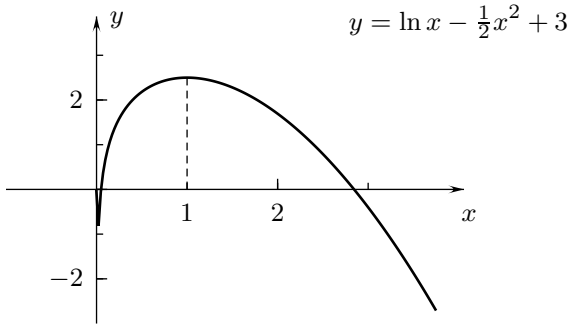


Figure A1

Example 2 **(a) What is the global maximum of $\mathbf{y = \ln x - \frac{1}{2}x^2 + 3}$?**
(b) Does the function have a global minimum?

Answer: [Global maximum] = 2.5 • (The graph is in Figure A2.) **(b)** There is no global minimum.

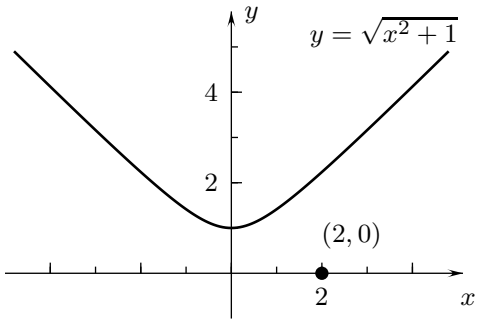


Figre A2

[†]Lecture notes to accompany Sections 4.3 and 4.5 of *Calculus* by Hughes-Hallett et al.

Example 3 **(a) Find the point on the curve $y = \sqrt{x^2 + 1}$ in Figure 1 that is closest to the point (2,0) . (Minimize the square of the distance.) (b) How far is (2,0) from the curve?**

FIGURE 1



Answer: (a) Figure A3a • The closest point on the curve is $(1, \sqrt{2})$ (Figure A3b)
(b) Its distance to $(2, 0)$ is $\sqrt{5}$.

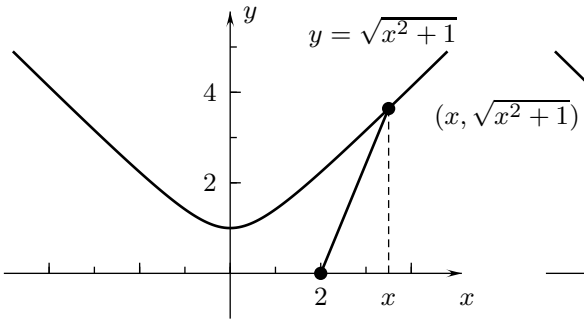


Figure A3a

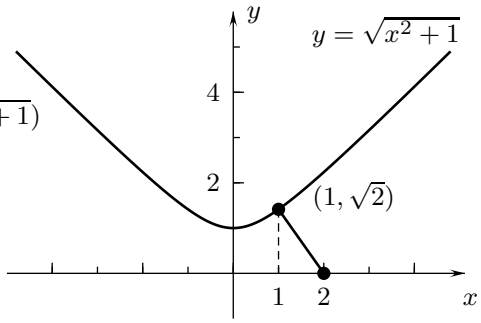


Figure A3b

Example 4 **Imagine you want to make a rectangular garden using a wall as one side and 40 feet of fence for the three other sides. What dimensions would give the garden the maximum area?**

Answer: Figure A4a • The area is a maximum for $w = 10$ and $L = 20$. • (These results are corroborated by the graph of the area as a function of the width in Figure A4b.)

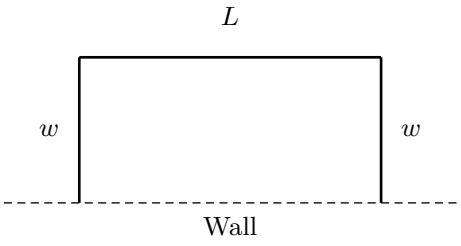


Figure A4a

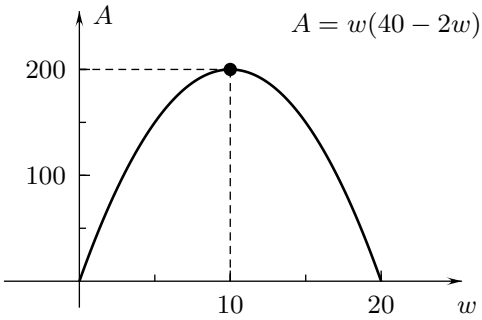


Figure A4b

Example 5 **A rectangular box with a square base and no top is to be constructed so that it has a volume of 4 cubic meters. How wide and how tall should the box be to minimize the total area of the base and four sides?**

Answer: Figure A5a • The box should be 2 meters wide and 1 meter high • (These results are corroborated by the graph of the area as a function of the width in Figure A5b.)

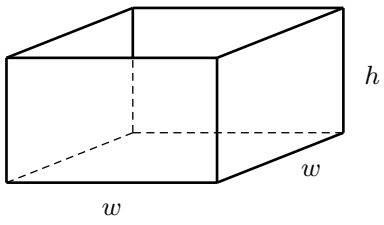


Figure A5a

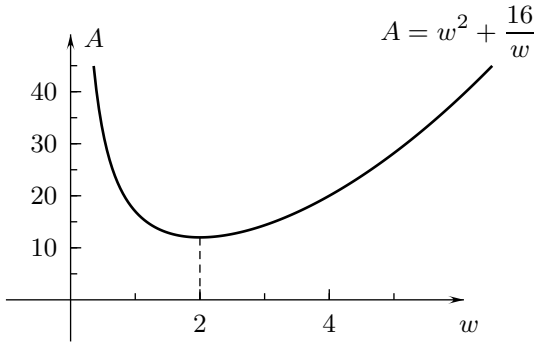


Figure A5b

Example 6 **A store will sell $\frac{1000}{25 + x^2}$ T-shirts if they charge x dollars per shirt ($1 \le x \le 20$). What price would maximize the revenue from the shirts?**

Answer: The revenue is maximized if the price is \$5 per shirt.

Interactive Examples

Work the following Interactive Examples on Shenk's web page, http//www.math.ucsd.edu/~ashenk/:[‡]

Section 4.5: Examples 1 and 2

[‡]The chapter and section numbers on Shenk's web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.