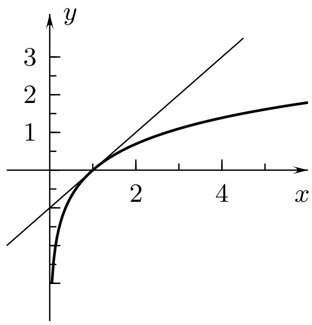
Math 10B. Lecture Examples.

Section 10.1. Taylor polynomials†

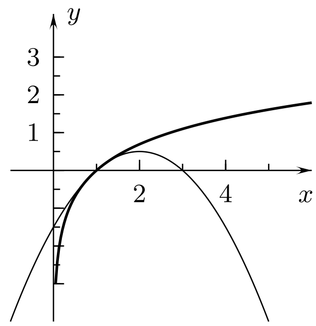
Example 1 Find the first-, second-, and third-degree Taylor polynomial approximations of $y = \ln x$, centered at $x = 1$.

Answer: $P_1(x) = x - 1$ • $P_2(x) = (x-1) - \frac{1}{2}(x-1)^2$ • $P_3(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$ • Figures A1a, A1b and A1c show $y = \ln x$ (the heavy curve) and the Taylor polynomials (the finer curves).



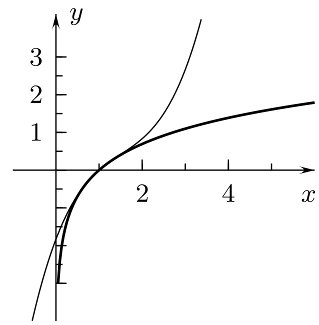
$P_1(x) = x - 1$

Figure A1a



$P_2(x) = (x-1) - \frac{1}{2}(x-1)^2$

Figure A1b



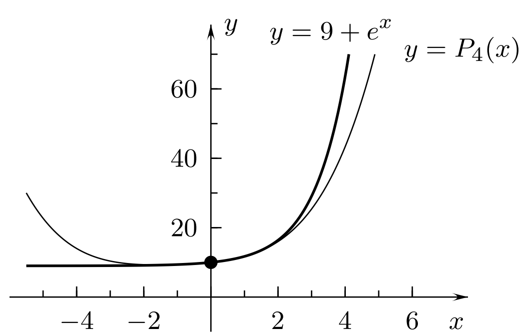
$P_3 = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$

Figure A1c

Example 2 **(a)** Find the fourth-degree Taylor Polynomial approximation $P_4(x)$ of $f(x) = e^x$ centered at $x = 0$. **(b)** How accurately does the polynomial $P_4(x)$ from part (a) approximate $9 + e^x$ at $x = 0.1$ and $x = 4$?

Answer: **(a)** $P_4(x) = 10 + x + \dfrac{1}{2!}x^2 + \dfrac{1}{3!}x^3 + \dfrac{1}{4!}x^4$ • (The graphs of $y = 9 + e^x$ and $y = P_4(x)$ are shown in Figure A2.) **(b)** $|(9 + e^x) - P_4(x)| \doteq 8.47 \times 10^{-8}$ at $x = 0.1$ and $\doteq 20.26$ at $x = 4$

Figure A2



†Lecture notes to accompany Section 10.1 of *Calculus* by Hughes-Hallett et al

Example 3 **(a)** Find the second-degree Taylor polynomial approximation $P_2(x)$ of $f(x) = x^2$ centered at $x = 0$. **(a)** Show that in this case $P_2(x) = f(x)$.

Answer: (a) $P_2(x) = 1 + 2(x-1) + (x-1)^2$ **(b)** $1 + 2(x-1) + (x-1)^2 = 1 + 2x - 2 + x^2 - 2x + 1 = x^2$

Interactive Examples

Work the following Interactive Examples on Shenk's web page, http//www.math.ucsd.edu/˜ashenk/:[‡]

Section 10.6: Examples 1–3, 4a

[‡]The chapter and section numbers on Shenk's web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.