(9/1/08)

Math 10B. Lecture Examples.

Section 10.3. Finding and using Taylor series[†]

Example 1 Use the Taylor series $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$ for $y = \sin x$ centered at $x = 0$ to give the Taylor series centered at $x = 0$ for $y = \sin(2x)$.

Answer: $\sin(2x) = \sum_{n=0}^{\infty} \frac{2^{2n+1}(-1)^n}{(2n+1)!} x^{2n+1}$

Example 2 Use the Taylor series

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n = x - \tfrac{1}{2}x^2 + \tfrac{1}{3}x^3 - \tfrac{1}{4}x^4 + \cdots$$

to find the Taylor series of $f(x) = x^2 \ln(1+x)$ centered at $x = 0$.

Answer: $x^2 \ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^{n+2} = x^3 - \tfrac{1}{2}x^4 + \tfrac{1}{3}x^5 - \tfrac{1}{4}x^6 + \cdots$

Example 3 What is the Taylor series centered at $x = 0$ for $y = f'(x)$ if $f(x) = \sum_{n=0}^{\infty} \frac{1}{2^n + 10} x^n$ for $-2 \le x < 2$?

Answer: $f'(x) = \sum_{n=1}^{\infty} \frac{n}{2^n + 10} x^{n-1}$

Example 4 Give an infinite series that equals $\int_0^1 f(x)\ dx$ where $f(x) = \sum_{n=0}^{\infty} \frac{1}{2^n + 10} x^n$ for $-2 \le x < 2$.

Answer: $\int_0^1 f(x)\,dx = \sum_{n=0}^{\infty} \frac{1}{(n+1)(2^n + 10)}$

Interactive Examples

Work the following Interactive Examples on Shenk's web page, http//www.math.ucsd.edu/˜ashenk/:[‡]

Section 10.7: Examples 5–9

[†]Lecture notes to accompany Section 10.3 of *Calculus* by Hughes-Hallett et al.

[‡]The chapter and section numbers on Shenk's web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.