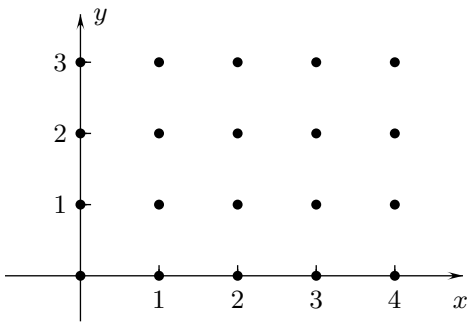
(9/1/08)

Math 10B. Lecture Examples.

Section 11.2. Slope fields[†]

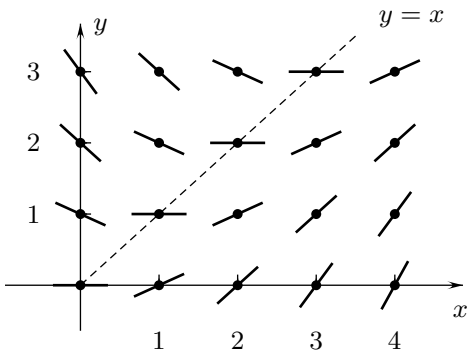
Example 1 **(a)** Draw the slope lines for the differential equation $\frac{dy}{dx} = \frac{1}{2}(x - y)$ at the twenty points with coordinates $x = 0, 1, 2, 3, 4$ and $y = 0, 1, 2, 3$ in Figure 1. **(b)** Describe the patterns of the slope lines and explain how they are determined by the differential equation.

FIGURE 1



Answer: (a) Figure A1 **(b)** One description and explanation: the slope lines are horizontal on the line $y = x$ where $\frac{1}{2}(x - y)$ is zero, point up to the right under the line $y = x$ where $y < x$ and $\frac{1}{2}(x - y)$ is positive, point down to the right above the line $y = x$ where $y > x$ and $\frac{1}{2}(x - y)$ is negative, and become steeper as they move away from the line $y = x$.

Figure A1



[†]Lecture notes to accompany Section 11.2 of *Calculus* by Hughes-Hallett et al

Example 2 Figure 3 shows the slope field for

$$\frac{dy}{dx} = 1$$

which consists of line segments of slope 1. Figure 2 shows the graphs of seven solutions of the differential equation. Find a formula for all solutions.

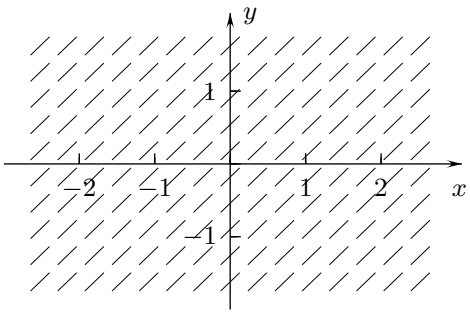


FIGURE 3

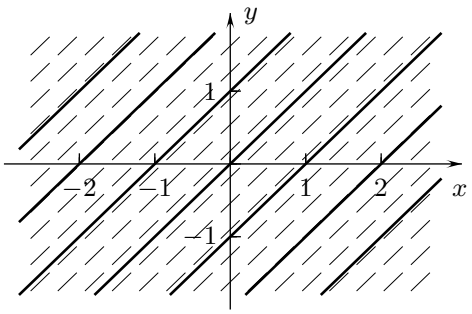


FIGURE 4

Answer: The solutions are $y = x + C$ with arbitrary constants C. (Their graphs are lines of slope 1.)

Example 3 Figure 5 shows the slope field for

$$\frac{dy}{dx} = x.$$

(a) Use the differential equation to explain how the slopes depend on the values of x and y. **(b)** Figure 6 shows the graphs of six solutions. Find a formula for all solutions.

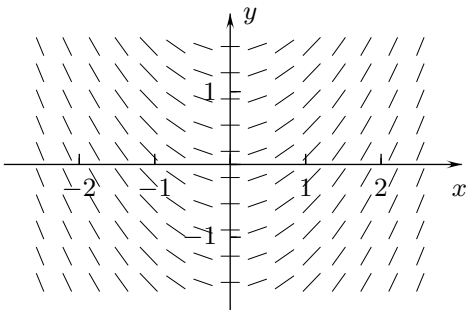


FIGURE 5

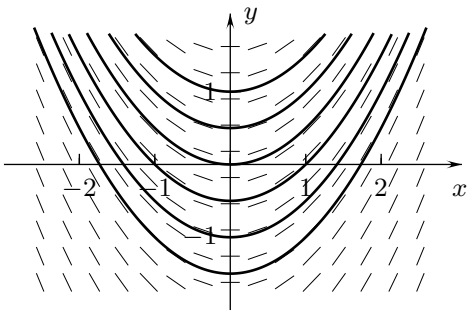


FIGURE 6

Answer: (a) The lines in each vertical column of Figure 5, where x is constant, are parallel because there is no y on the right side of the differential equation. • The lines are horizontal along the y-axis, where $x = 0$, have positive slope to the right of the y-axis, where x is positive, have negative slope to the left of the y-axis, where x is negative, and get steeper as x moves away from 0 in either direction.
(b) The solutions are $y = \frac{1}{2}x^2 + C$ with arbitrary constants C. (Their graphs are parabolas.)

Example 4 The slope field for $\dfrac{dy}{dx} = h(x, y)$ is in Figure 7. Draw the approximate graphs of the solutions with the initial values **(a)** $y(0) = 1$ and **(b)** $y(0) = 4$.

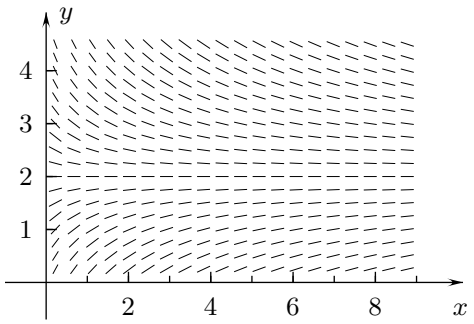


FIGURE 7

Answer: Figure A4

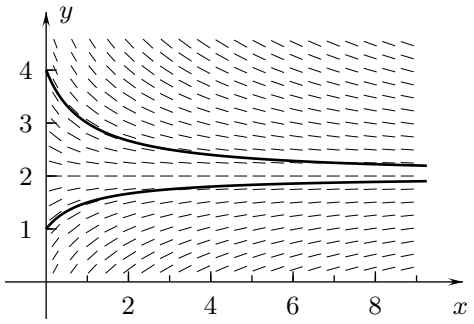


Figure A4

Example 5 Draw the approximate graph of the solution of the initial-value problem $\frac{dy}{dx} = K(x, y), y(0) = 2$. The slope field for the differential equation is in Figure 8.

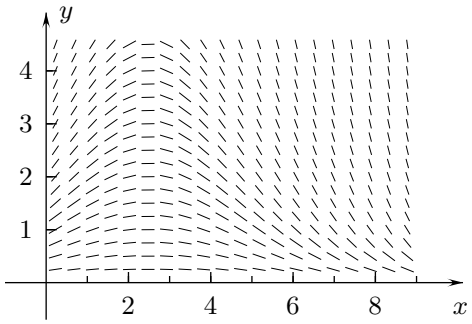


FIGURE 8

Answer: Figure A5

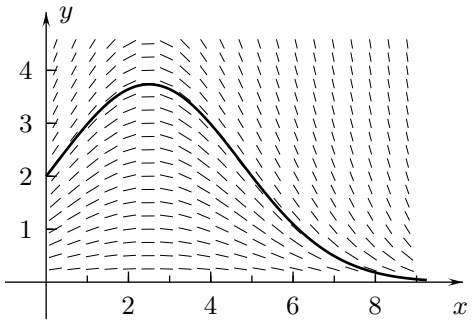


Figure A5

Interactive Examples

Work the following Interactive Examples on Shenk's web page, http//www.math.ucsd.edu/˜ashenk/:[‡]

Section 9.1: Example 4

[‡]The chapter and section numbers on Shenk's web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.