(9/8/08)

Math 10B. Lecture Examples.

Section 11.4. Separation of variables†

Example 1 Figure 1 shows the slope field of the differential equation

$$\frac{dy}{dx} = y$$

and Figure 2 shows the graphs of eight solutions. **(a)** Use the differential equation to explain the pattern of the slope lines. **(b)** Find an equation for all solutions.

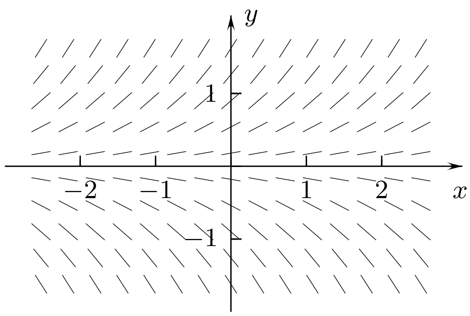


FIGURE 1

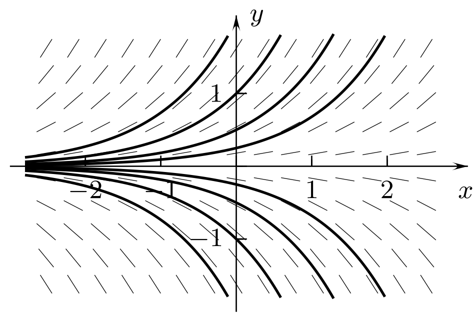


FIGURE 2

Answer: (a) One description and explanation: The lines in the slope field of $\frac{dy}{dx} = y$ in Figure 1 have the same slope along each horizontal line because the formula on the right does not involve x. • The lines are horizontal along the x-axis where $y = 0$, have positive slopes above the x-axis where $y > 0$, and have negative slopes below the x-axis where $y < 0$, and they become steeper as y increases through positive values or decreases through negative values.
(b) The solutions are $y = Ce^x$ with arbitrary constants C.

Example 2 Find the solution of the initial-value problem $\frac{dy}{dx} = 2y\cos x$, $y(0) = 4$.

Answer: $y = 4e^{2\sin x}$

Example 3 Check the result of Example 2.

Answer: Set $y = 4e^{2\sin x}$. • $y(0) = 4e^{2\sin(0)} = 4$ • The initial condition is satisfied. •
$\frac{dy}{dx} = \frac{d}{dx}(4e^{2\sin x}) = 8(\cos x)e^{2\sin x} = 2y\cos x$ • The differential equation is satisfied.

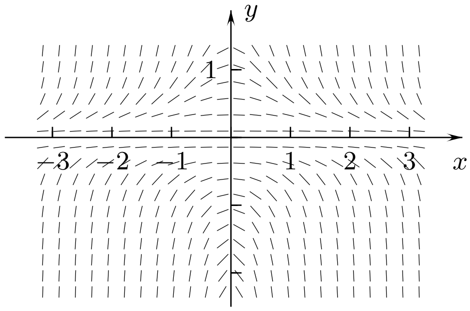
Example 4 Find the solutions of the differential equation

$$\frac{dy}{dx} = -2xy^2$$

with the initial conditions **(a)** $y(0) = 1$ • $y(0) = -\frac{1}{4}$.

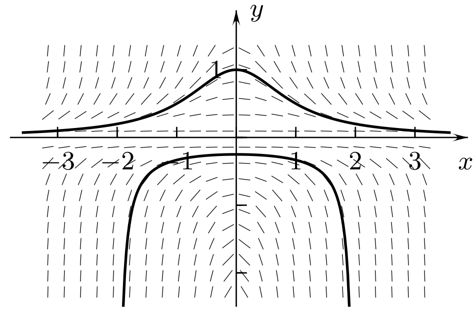
Answer: (a) $y = \frac{1}{x^2+1}$ **(b)** $y = \frac{1}{x^2-4}$ • (Figure A4a shows the slope field for the differential equation **(6)**, and Figure A4b gives the graphs of the solutions.)

†Lecture notes to accompany Section 11.4 of *Calculus* by Hughes-Hallett et al



Slope field of $\dfrac{dy}{dx} = -2xy^2$

Figure A4a



Solutions with $y(0) = 1$ and $y(0) = -\frac{1}{4}$

Figure A4b

Example 5 Solve the initial-value problem $K'(x) = \sqrt{xK(x)},\ K(1) = 1$

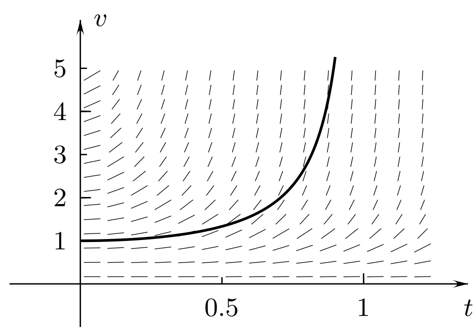
Answer: $K = (\frac{1}{3}x^{3/2} + \frac{2}{3})^2$

Example 6 Find all nonzero solutions of $\dfrac{dQ}{dx} = -3Q^{1/4}$.

Answer: $Q = (C - \frac{9}{4}x)^{4/3}$

Example 6 **(a)** A two-gram object is moving on an s-axis with distances measured in centimeters. Its velocity in the positive direction is 1 centimeter per second at time $t = 0$ (seconds) and the force on it at time $t > 0$ is $4tv^2$ dynes in the positive s-direction if its velocity is v centimeters per second at that time. Give an initial-value problem satisfied by $v = v(t)$. **(b)** Give a formula for v for $t \geq 0$. **(c)** What happens to the velocity as $t \to \infty$? (The slope field and graph of the solution are in Figure 3.)

FIGURE 3



Answer: (a) Initial-value problem: $\dfrac{dv}{dt} = 2tv^2, v(0) = 1$ **(b)** $v = \dfrac{1}{1 - t^2}$ **(c)** $v \to \infty$ as $t \to 1^-$

Interactive Examples

Work the following Interactive Examples on Shenk's web page, http//www.math.ucsd.edu/˜ashenk/:[‡]

Section 9.1: Examples 1–3, 5, 6, 8

[‡]The chapter and section numbers on Shenk's web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.