Math 10B. Lecture Examples.

Section 5.3. The Fundamental Theorem and interpretations[†]

Example 1   What is the value of $\int_0^2 \mathbf{F}'(\mathbf{x}) \, d\mathbf{x}$ if $\mathbf{F}(\mathbf{x}) = \mathbf{x}^4 + 5$?

Answer: $\int_0^2 F'(x) \, dx = 16$

Example 2   Figure 1 shows the graph of the continuous derivative $\mathbf{r} = \mathbf{G}'(\mathbf{x})$ of a continuous function $\mathbf{y} = \mathbf{G}(\mathbf{x})$. Region **A** in the drawing has area **89** and region **B** has area **62**. What is $\mathbf{G(6)} - \mathbf{G(1)}$?
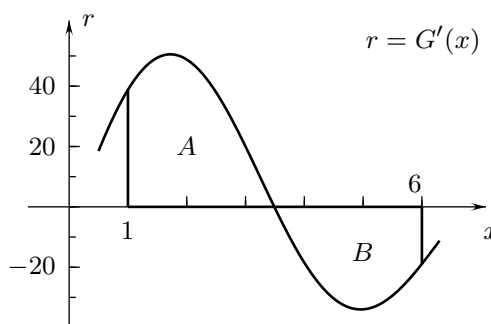

FIGURE 1

Answer: $G(6) - G(1) = 27$

Example 3   A tank contains 100 gallons of water at $\mathbf{t = 0}$ (minutes) and water is added at the rate of **20t** gallons per minute at time **t** for $\mathbf{0 \leq t \leq 4}$. How much water is in the tank at $\mathbf{t = 4}$?

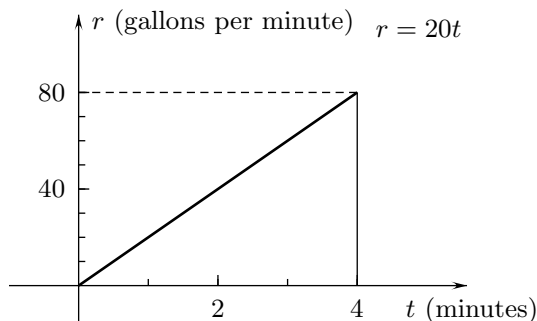**Answer:** Figure A3 ● The tank has 260 gallons of water in it at $t = 4$

Figure A3

[†]Lecture notes to accompany Section 5.3 of *Calculus* by Hughes-Hallett et al.

Example 4   **An object moving on an *s*-axis with coordinates given in feet is at s = 10 at time t = 1. The graph of its velocity v(t) = s′(t) in the positive s-direction is shown in Figure 2. The area of region A in the drawing is 16.5 and the area of region B is 6.75. Where is the object at t = 4?**
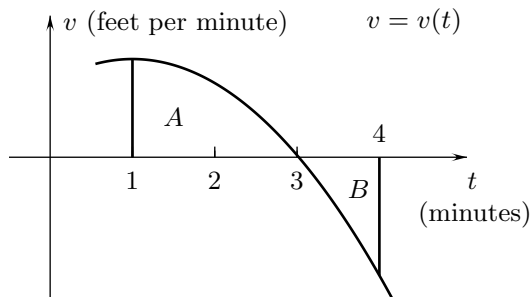

FIGURE 2

Answer: $s(4) = 19.75$

Example 5   **The table below lists the rate r = r(t) at which residents of the U.S. spent money on commodities and services, as measured on January 1 every other year just before and during the Great Depression. Give an estimate of the total spent from the beginning of 1929 to the beginning of 1939**

Rate of spending (billion dollars per year)

t	1929	1931	1933	1935	1937	1939
r(t)	77.2	60.5	45.8	55.7	66.5	72.0

Answer: [Total spent from the beginning of 1929 to the beginning of 1939] $= \displaystyle\int_{1929}^{1939} r(t)\, dt \approx 611.4$ billion dollars

with a left Riemann sum (Figure A5a) or ≈ 601 billion dollars with a right Riemann sum (Figure A5b)
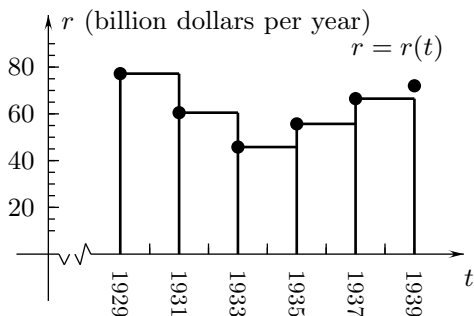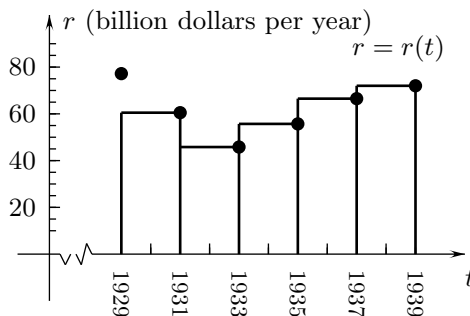

Figure A5a

Figure A5b

Example 6   **Figure 3 shows the graph of blood pressure $p = p(t)$ at the opening of the aorta leading from a person's heart during about two and a half heart beats. The person's heart beats once every 0.8 seconds, so $p = p(t)$ is periodic of period 0.8 seconds. Estimate the person's average blood pressure during one period.**
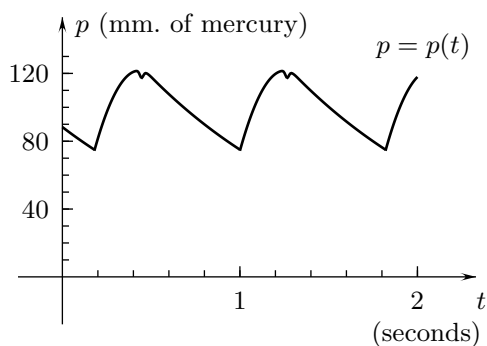
FIGURE 3

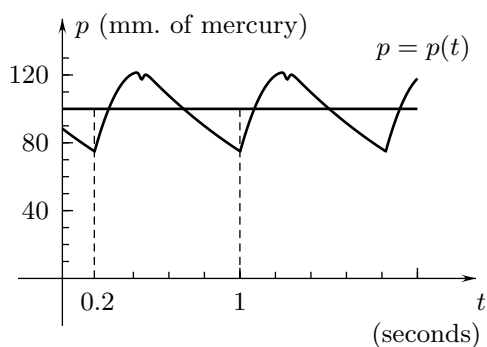**Answer:** Figure A6 ● The average blood pressure is about 100 millimeters of mercury.

Figure A6

Interactive Examples

Work the following Interactive Examples on Shenk's web page, http//www.math.ucsd.edu/˜ashenk/:[‡]

    Section 6.3: Examples 1–4

    Section 6.6: Example 1

    Section 7.7: Example 2

[‡]The chapter and section numbers on Shenk's web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.