Math 10B. Lecture Examples.

Section 6.2. Constructing antiderivatives analytically†

Example 1  (a) Find the antiderivative \( \int \left( 3\sqrt{x} + \frac{4}{x^2} - 3 \right) \, dx \).

(b) Check the answer by differentiation.

Answer: (a) \( \int \left( 3\sqrt{x} + \frac{4}{x^2} - 3 \right) \, dx = 2x^{3/2} - 4x^{-1} - 3x + C \)

(b) \( \frac{d}{dx}(2x^{3/2} - 4x^{-1} - 3x) = 3\sqrt{x} + \frac{4}{x^2} - 3 \)

Example 2  What is the value of the integral \( \int_{-1}^{1} x^2 \, dx \)?

Answer: \( \int_{-1}^{1} x^2 \, dx = \frac{2}{3} \)

Example 3  Evaluate \( \int_{1}^{2} (4x^{1/3} + 6x^{-2}) \, dx \).

Answer: \( \int_{1}^{2} (4x^{1/3} + 6x^{-2}) \, dx = 3(2^{4/3}) \)

Example 4  Find the area of the region bounded by the curve \( y = 3x^2 - x^3 \) and the x-axis.

Answer: Figure A3 • [Area] = \( \frac{27}{4} \)

![Figure A3](image)

Example 5  Suppose that the temperature in a room is 50°F at time \( t = 0 \) (hours) and that the rate of change of the temperature is \( r = 12t^2 - 4t^3 \) degrees per hour at time \( t \) for \( 0 \leq t \leq 2 \). What is the temperature at \( t = 2 \)?

Answer: The temperature at \( t = 2 \) is 66°F

†Lecture notes to accompany Section 6.2 of Calculus by Hughes-Hallett et al.
Example 6  Find the area of the region bounded by \( y = x^2 \) and \( y = 2x \).

Answer: Figure A5. \([\text{Area}] = \frac{4\pi}{3}\)

![Figure A5](image)

Example 7  Find the area of the region bounded by \( y = \sin x \) and \( y = 2 \) for \( 0 \leq x \leq \frac{1}{2}\pi \).

Answer: \([\text{Area}] = \pi - 1\)

Example 8  Evaluate \( \int_{-2}^{2} \frac{1}{x} \, dx \) and \( \int_{-5}^{5} \frac{1}{x} \, dx \).

Answer: \( \int_{2}^{5} \frac{1}{x} \, dx = \ln(5) - \ln(2) \cdot \int_{-5}^{-2} \frac{1}{x} \, dx = \ln(2) - \ln(5) \)

Example 8  Find a formula for the function \( y = g(x) \) such that \( g'(x) = e^x \) for all \( x \) and \( g(2) = 10 \).

Answer: \( g(x) = e^x + 10 - e^2 \)

Example 9  A car is 30 miles north of a town at time \( t = 0 \) (hours) and its velocity toward the north is \( v(t) = 60 + 5 \cos t + 8 \sin t \) miles per hour for \( 0 \leq t \leq 3 \).

Where is it at \( t = 3 \)?

Answer: The car is \( 5 \sin(3) - 8 \cos(3) + 218 \approx 226.63 \) miles north of the town at \( t = 3 \).

Interactive Examples

Work the following Interactive Examples on Shenk’s web page, \( \text{http://www.math.ucsd.edu/~ashenk/}^{\dagger} \):

Section 6.5: 1–4
Section 6.7: 1–3, 8, and 9
Section 7.1: 1 and 2
Section 7.7: 1 and 3

\[^{\dagger}\text{The chapter and section numbers on Shenk’s web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.}\]