Math 10B. Lecture Examples.

Section 6.2. Constructing antiderivatives analytically†

Example 1 **(a) Find the antiderivative** $\displaystyle\int \left(\mathbf{3\sqrt{x} + \frac{4}{x^2} - 3}\right)\, \mathbf{dx}$**.**

(b) Check the answer by differentiation.

Answer: (a) $\displaystyle\int \left(3\sqrt{x} + \frac{4}{x^2} - 3\right)\, dx == 2x^{3/2} - 4x^{-1} - 3x + C$

(b) $\displaystyle\frac{d}{dx}(2x^{3/2} - 4x^{-1} - 3x) = 3\sqrt{x} + \frac{4}{x^2} - 3$

Example 2 **What is the value of the integral** $\displaystyle\int_{-1}^{1} \mathbf{x^2\, dx}$**?**

Answer: $\displaystyle\int_{-1}^{1} x^2\, dx = \tfrac{2}{3}$

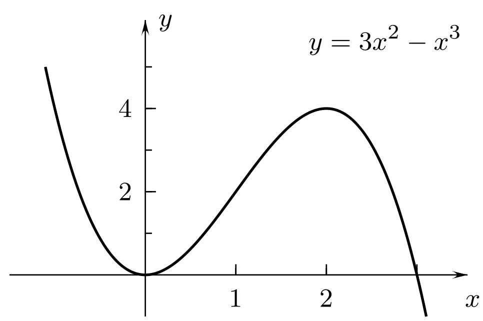
Example 3 **Evaluate** $\displaystyle\int_{1}^{2} (\mathbf{4x^{1/3} + 6x^{-2}})\, \mathbf{dx}$**.**

Answer: $\displaystyle\int_{1}^{2} (4x^{1/3} + 6x^{-2})\, dx = 3(2^{4/3})$

Example 4 **Find the area of the region bounded by the curve** $\mathbf{y = 3x^2 - x^3}$ **and the x-axis.**

Answer: Figure A3 ● [Area] $= \frac{27}{4}$

Figure A3



Example 5 **Suppose that the temperature in a room is** $\mathbf{50^\circ F}$ **at time** $\mathbf{t = 0}$ **(hours) and that the rate of change of the temperature is** $\mathbf{r = 12t^2 - 4t^3}$ **degrees per hour at time** $\mathbf{t}$ **for** $\mathbf{0 \le t \le 2}$**. What is the temperature at** $\mathbf{t = 2}$**?**

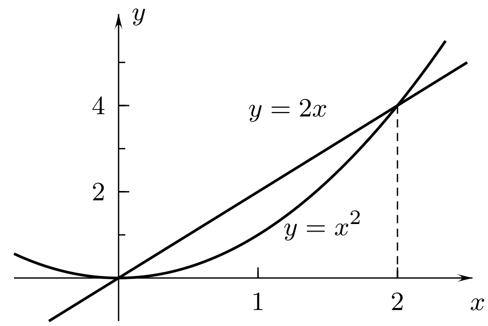
Answer: The temperature at $t = 2$ is 66°F

†Lecture notes to accompany Section 6.2 of *Calculus* by Hughes-Hallett et al.

Example 6 **Find the area of the region bounded by $y = x^2$ and $y = 2x$.**

Answer: Figure A5. [Area] $= \frac{4}{3}$

Figure A5



Example 7 **Find the area of the region bounded by $y = \sin x$ and $y = 2$ for $0 \le x \le \frac{1}{2}\pi$.**

Answer: [Area] $= \pi - 1$

Example 8 **Evaluate $\int_2^5 \frac{1}{x}\,dx$ and $\int_{-5}^{-2} \frac{1}{x}\,dx$.**

Answer: $\int_2^5 \frac{1}{x}\,dx = \ln(5) - \ln(2)$ • $\int_{-5}^{-2} \frac{1}{x}\,dx = \ln(2) - \ln(5)$

Example 8 **Find a formula for the function $y = g(x)$ such that $g'(x) = e^x$ for all x and $g(2) = 10$.**

Answer: $g(x) = e^x + 10 - e^2$

Example 9 **A car is 30 miles north of a town at time $t = 0$ (hours) and its velocity toward the north is $v(t) = 60 + 5\cos t + 8\sin t$ miles per hour for $0 \le t \le 3$. Where is it at $t = 3$?**

Answer: The car is $5\sin(3) - 8\cos(3) + 218 \doteq 226.63$ miles north of the town at $t = 3$.

Interactive Examples

Work the following Interactive Examples on Shenk's web page, http//www.math.ucsd.edu/˜ashenk/:[‡]

Section 6.5: 1–4

Section 6.7: 1–3, 8, and 9

Section 7.1: 1 and 2

Section 7.7: 1 and 3

[‡]The chapter and section numbers on Shenk's web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.