(9/30/08)

Math 10B. Lecture Examples.

Section 7.4. Algebraic identities and trigonometric substitutions[†]

Example 1 **Find the partial-fraction decomposition of $\mathbf{\frac{1}{x(x-1)}}$. Check by giving the result a common denominator.**

Answer: $\frac{1}{x(x-1)} = \frac{1}{x-1} - \frac{1}{x}$ • Check: $\frac{1}{x-1} - \frac{1}{x} = \frac{x}{x(x-1)} - \frac{x-1}{x(x-1)} = \frac{x-(x-1)}{x(x-1)} = \frac{1}{x(x-1)}$

Example 2 **Use the result of Example 1 to find the function $\mathbf{G(x)}$ such that $\mathbf{G'(x) = \frac{1}{x(x-1)}}$ and $\mathbf{G(2) = 0}$.**

Answer: $G(x) = \ln|x-1| - \ln|x| + \ln(2)$

Example 3 **Find the partial-fraction decomposition of $\mathbf{y = \frac{3x^2-1}{x^3-x}}$.**

Answer: $\frac{3x^2-1}{x^3-x} = \frac{1}{x} + \frac{1}{x-1} + \frac{1}{x+1}$

Example 4 **Use the result $\mathbf{\frac{3x^2-1}{x^3-x} = \frac{1}{x} + \frac{1}{x-1} + \frac{1}{x+1}}$ of Example 3 to perform the integration, $\mathbf{\int \frac{3x^2-1}{x^3-x}\,dx}$.**

Answer: $\int \frac{3x^2-1}{x^3-x}\,dx = \ln|x| + \ln|x-1| + \ln|x+1| + C$

Example 5 **Evaluate $\mathbf{\int_1^2 \frac{x^3+1}{x^2}\,dx}$.**

Answer: $\int_1^2 \frac{x^3+1}{x^2}\,dx = 2$

Example 6 **Find the antiderivative $\mathbf{\int \frac{3x^3+12x+1}{x^2+4}\,dx}$.**

Answer: Figure A6 • $\frac{3x^3+12x+1}{x^2+4} = 3x + \frac{1}{x^2+4}$ • $\int \frac{3x^3+12x+1}{x^2+4}\,dx = \frac{3}{2}x^2 + \frac{1}{2}\tan^{-1}\left(\frac{1}{2}x\right) + C$

$$\begin{array}{r|l} & 3x \\ \hline x^2+4 & 3x^3 + 12x + 1 \\ & 3x^3 + 12x \\ \hline & \qquad\qquad\quad 1 \end{array}$$

Figure A6

Interactive Examples

Work the following Interactive Examples on Shenk's web page, http//www.math.ucsd.edu/˜ashenk/:[‡]

Section 8.4: Examples 1 through 4

[†] Lecture notes to accompany Section 7.4 of *Calculus* by Hughes-Hallett et al.

[‡] The chapter and section numbers on Shenk's web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.