(9/1/08)

Math 10B. Lecture Examples.

Section 9.2. Geometric series[†]

Example 1 Suppose you want to go from a point A toward a second point B two miles away. First you go one mile (Figure 1). Then you go a half mile further for a total of $1 + \frac{1}{2}$ miles (Figure 2). Next you go half that distance (Figure 3), and so forth, so that at each stage you go half as far as you did in the previous stage. **(a)** How far have you gone after one, two, three, five, and eight stages? **(b)** Predict the limit of your distance from A as the number of stages tends to ∞.

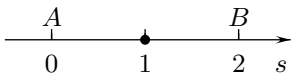

FIGURE 1

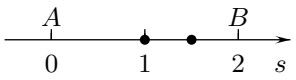

FIGURE 2

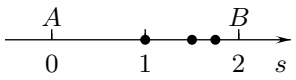

FIGURE 3

Answer: (a) See the table below. **(b)** The distance seems to be approaching $\dfrac{1}{1-\frac{1}{2}} = 2$ miles.

Stages	Distance (miles)	Decimal approximation
1	1	1
1–2	$1 + \frac{1}{2}$	1.5
1–3	$1 + \frac{1}{2} + (\frac{1}{2})^2$	1.75
1–5	$1 + \frac{1}{2} + (\frac{1}{2})^2 + (\frac{1}{2})^3 + (\frac{1}{2})^4$	1.9375
1–8	$1 + \frac{1}{2} + (\frac{1}{2})^2 + (\frac{1}{2})^3 + (\frac{1}{2})^4 + (\frac{1}{2})^5 + (\frac{1}{2})^6 + (\frac{1}{2})^7$	1.9921875

[†]Lecture notes to accompany Section 9.2 of *Calculus* by Hughes-Hallett et al.

Example 2 This time suppose you go one mile from A toward B in the first stage, but then $\frac{1}{4}$ mile back toward A in the second stage, $(\frac{1}{4})^2 = \frac{1}{16}$ mile away from A in the third stage, $(\frac{1}{4})^3 = \frac{1}{64}$ mile toward A in the fourth stage, and so on. **(a)** How far you are from A after one, two, three, five, and eight stages? **(b)** Predict the limit of your distance from A as the number of stages tends to ∞.

Answer: (a) See the table below. **(b)** The distance seems to be approaching $\dfrac{1}{1+\frac{1}{4}} = 0.8$.

Stages	Total distance	Decimal approximation
1	1	1
1–2	$1 - \frac{1}{4}$	0.75
1–3	$1 - \frac{1}{4} + (\frac{1}{4})^2$	0.8125
1–5	$1 - \frac{1}{4} + (\frac{1}{4})^2 - (\frac{1}{4})^3 + (\frac{1}{4})^4$	0.8007813
1–8	$1 - \frac{1}{4} + (\frac{1}{4})^2 - (\frac{1}{4})^3 + (\frac{1}{4})^4 - (\frac{1}{4})^5 + (\frac{1}{4})^6 - (\frac{1}{4})^7$	0.7999878

Example 3 Give a concise formula for $\displaystyle\sum_{n=2}^{511} (0.99)^n$ and find its approximate decimal value.

Answer: $\displaystyle\sum_{n=2}^{511} (0.99)^n = (0.99)^2 \left(\frac{1 - (0.99)^{510}}{1 - 0.99} \right) \doteq 97.427602$

Example 4 Evaluate $\displaystyle\sum_{j=0}^{400} (-1)^j$

Answer: $\displaystyle\sum_{j=0}^{400} (-1)^j = 1$

Example 5 Does the infinite geometric series $\sum_{j=0}^{\infty} (0.95)^j$ converge? If so, give its value.

Answer: The Geometric Series converges because $|0.95| < 1$. • $\sum_{j=0}^{\infty} (0.95)^j = 20$

Example 6 Give the exact value of the infinite geometic series $\sum_{j=3}^{\infty} (-\frac{3}{4})^j$.

Answer: $\sum_{j=3}^{\infty} (-\frac{3}{4})^j = -\frac{21}{112}$

Example 7 A woman opens a new savings account on January 1, 2000 and deposits \$5000 in it. She deposits \$5000 on each subsequent January 1 through the year 1020. The account pays 8% annual interest compounded annually, based on the December 31 balance. She does not make any additional withdrawals or deposits. How much is in the account on January 2, 2020?

Answer: [Balance] $= 5000 \left(\dfrac{1 - (1.08)^{21}}{1 - 1.08} \right) \doteq \$252{,}115$

Interactive Examples

Work the following Interactive Examples on Shenk's web page, http//www.math.ucsd.edu/~ashenk/:[‡]

Section 10.2: Examples 1–3

[‡]The chapter and section numbers on Shenk's web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.