Math 10B. Lecture Examples.
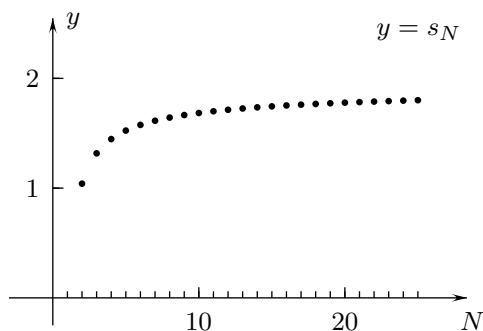
Section 9.3. Convergence of series[†]

Example 1   Does $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ converge or diverge?

Answer: $\displaystyle\int_2^{\infty} \frac{1}{x(\ln x)^2} = \frac{1}{\ln(2)}$ ● The improper integral and the infinite series converge. (The first 24 partial sums of the series are plotted in Figure A1.)

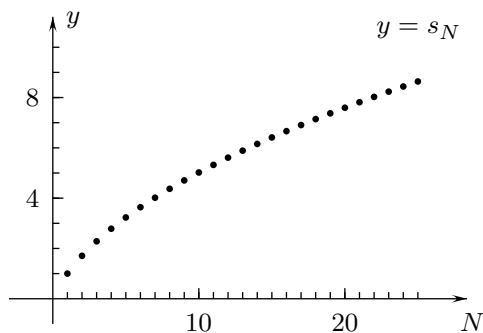$$s_N = \sum_{n=2}^{N} \frac{1}{n(\ln n)^2}$$

Figure A1

Example 2   Does $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ converge or diverge?

Answer: $\displaystyle\int_1^{\infty} \frac{1}{\sqrt{x}}\, dx = \infty$ ● The infinite series diverges. (Its first 25 partial sums are plotted in Figure A2.)

$$s_N = \sum_{n=1}^{N} \frac{1}{\sqrt{n}}$$

Figure A2

Example 3   Does $\displaystyle\sum_{n=1}^{\infty} ne^{-n^2}$ converge?

Answer: $\displaystyle\int_1^{\infty} xe^{-x^2}\, dx = \tfrac{1}{2}e^{-1}$ ● $\displaystyle\sum_{n=1}^{\infty} ne^{-n^2}$ converges.

Example 4   Does $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1.75}}$ converge or diverge?

Answer: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1.75}}$ converges.

[†]Lecture notes to accompany Section 9.3 of *Calculus* by Hughes-Hallett et al

Example 5   Does $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+1}$ converge or diverge?

        Answer: $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+1}$ diverges.

Interactive Examples

Work the following Interactive Examples on Shenk's web page, http//www.math.ucsd.edu/~ashenk/:[‡]

    Section 10.3: Examples 1–4

[‡]The chapter and section numbers on Shenk's web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.