(9/1/08)

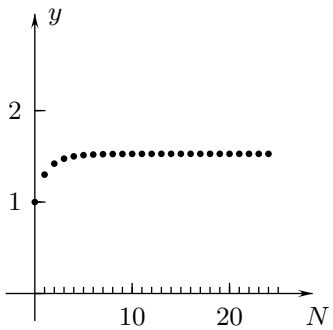
Math 10B. Lecture Examples.

Section 9.4. Tests for convergence[†]

Example 1 Does the series $\displaystyle\sum_{n=0}^{\infty} \frac{(0.6)^n}{n+1}$ converge?

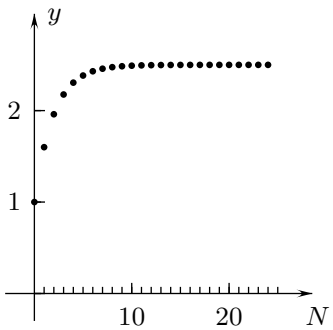
Answer: $\displaystyle\sum_{n=0}^{\infty} \frac{(0.6)^n}{n+1}$ converges by the Comparison Test with the convergent Geometric Series $\displaystyle\sum_{n=0}^{\infty} (0.6)^n$.

(All but the first partial sum of $\displaystyle\sum_{n=0}^{\infty} \frac{(0.6)^n}{n+1}$ in Figure A1a is less than the corresponding partial sum of $\displaystyle\sum_{n=0}^{\infty} (0.6)^n$ in Figure A1b.)



$$y = \sum_{n=0}^{N} \frac{(0.6)^n}{n+1}$$

Figure A1a



$$y = \sum_{n=0}^{N} (0.6)^n$$

Figure A1b

Example 2 Does $\displaystyle\sum_{n=1}^{\infty} \frac{10\cos(3n)}{n^{3/2}}$ converge?

Answer: $\displaystyle\sum_{n=1}^{\infty} \frac{10\cos(3n)}{n^{3/2}}$ converges.

Example 3 Does $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n-1}$ converge or diverge?

Answer: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n-1}$ diverges.

Example 4 Does $\displaystyle\sum_{n=0}^{\infty} \frac{2^n+10}{5^n}$ converge or diverge?

Answer: $\displaystyle\sum_{n=0}^{\infty} \frac{2^n+10}{5^n}$ converges by the Limit Comparison Test with the convergent geometric series $\displaystyle\sum_{n=0}^{\infty} \left(\tfrac{2}{5}\right)^n$.

[†]Lecture notes to accompany Section 9.4 of *Calculus* by Hughes-Hallett et al

Example 5 Apply the Ratio Test to $\displaystyle\sum_{n=0}^{\infty} \frac{5^n}{n!}$.

Answer: The series converges.

Example 6 Apply the Ratio Test to $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{n^3}$.

Answer: $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{n^3}$ diverges.

Example 7 Show that the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ converges.

Answer: The series converges by the Alternating Series Test.

Example 8 Does $\displaystyle\sum_{n=1}^{\infty} (-1)^n e^{1/n}$ converge?

Answer: No, the series diverges.

Interactive Examples

Work the following Interactive Examples on Shenk's web page, http//www.math.ucsd.edu/~ashenk/:[‡]

Section 10.4: Examples 1–5

Section 10.5: Examples 1–5

[‡]The chapter and section numbers on Shenk's web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.