

Math 10C. Lecture Examples.

Sections 13.1 and 13.2. Vectors[†]

Example 1 Find the x- and y-components of the vector \mathbf{u} of length 10 with angle of inclination $\frac{5}{6}\pi$.

Answer: Figure A1a • $\mathbf{u} = \langle -5\sqrt{3}, 5 \rangle$ • Figure A1b

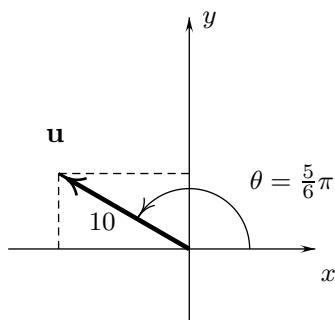


Figure A1a

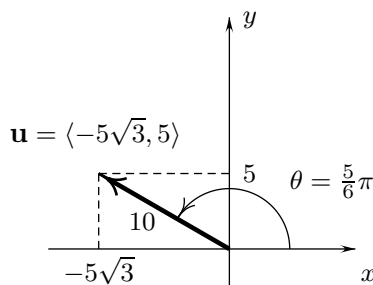


Figure A1b

Example 2 Find an angle of inclination of the vector $\mathbf{w} = \langle 3, 4 \rangle$.

Answer: Figure A2. • $\theta = \tan^{-1}(\frac{4}{3})$

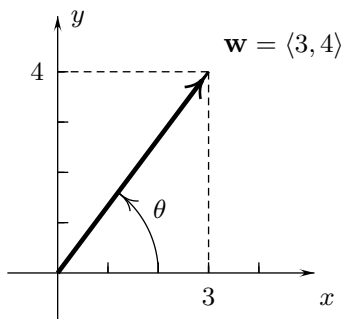


Figure A2

[†]Lecture notes to accompany Sections 13.1 and 13.2 of *Calculus* by Hughes-Hallett et al.

Example 3 Calculate (a) $\mathbf{v} + \mathbf{w}$ and (b) $\mathbf{w} - \mathbf{v}$ for $\mathbf{v} = \langle 4, 1 \rangle$ and $\mathbf{w} = \langle 1, 3 \rangle$. Then draw the four vectors.

Answer: (a) $\mathbf{v} + \mathbf{w} = \langle 5, 4 \rangle$ • Figure A3a (b) $\mathbf{w} - \mathbf{v} = \langle -3, 2 \rangle$ • Figure A3b

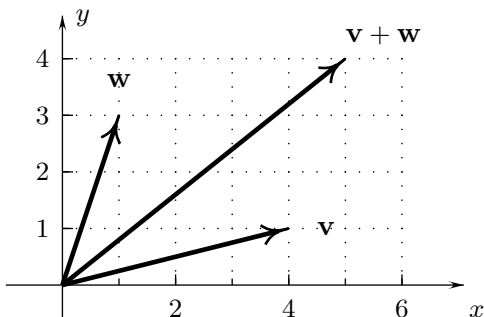


Figure A3a

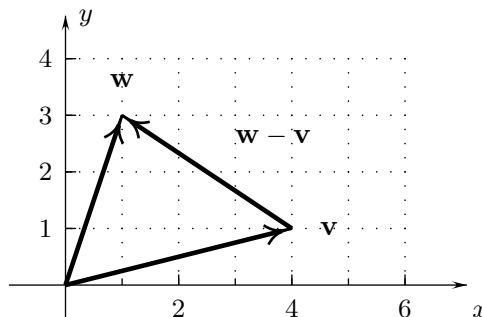


Figure A3b

Example 4 Write $3\langle 4, -1 \rangle - 2\langle 10, -5 \rangle$ in the form $\langle \mathbf{a}, \mathbf{b} \rangle$.

Answer: $3\langle 4, -1 \rangle - 2\langle 10, -5 \rangle = \langle -8, 7 \rangle$

Example 5 Give the unit vector \mathbf{e}_v and the vector \mathbf{w} of length 5 with the same direction as $\mathbf{v} = \langle -3, 2 \rangle$.

Answer: $\mathbf{e}_v = \frac{\langle -3, 2 \rangle}{\sqrt{13}}$ • $\mathbf{w} = \frac{\langle -15, 10 \rangle}{\sqrt{13}}$

Example 6 Express $3(4\mathbf{i} - \mathbf{j}) - 2(10\mathbf{i} - 5\mathbf{j})$ in the form $a\mathbf{i} + b\mathbf{j}$.

Answer: $3(4\mathbf{i} - \mathbf{j}) - 2(10\mathbf{i} - 5\mathbf{j}) = -8\mathbf{i} + 7\mathbf{j}$. (Example 4 is the same calculation with different notation.)

Example 7 One man is lifting a boulder with a rod while another is pulling it with a rope as in Figure 1. (a) Find the x- and y-components of the two force vectors, with the usual orientation of axes. (b) Find the resultant of the two forces and the approximate decimal values of its magnitude and angle of inclination

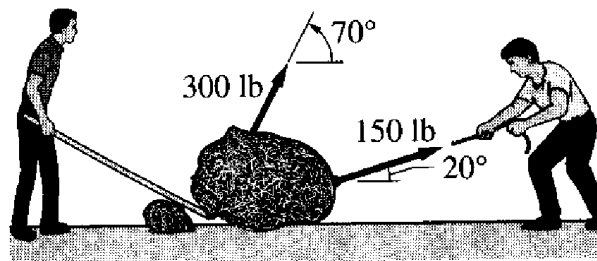


FIGURE 1

Answer: (a) [Force exerted by the man with the rod] = $\mathbf{F} = 300 \left\langle \cos \left(\frac{7}{18} \pi \right), \sin \left(\frac{7}{18} \pi \right) \right\rangle$ pounds. •

[Force exerted by the man with the rope] = $\mathbf{G} = 150 \left\langle \cos \left(\frac{1}{9} \pi \right), \sin \left(\frac{1}{9} \pi \right) \right\rangle$ pounds.

(b) [Resultant] = $\left\langle 300 \cos \left(\frac{7}{18} \pi \right) + 150 \cos \left(\frac{1}{9} \pi \right), 300 \sin \left(\frac{7}{18} \pi \right) + 150 \sin \left(\frac{1}{9} \pi \right) \right\rangle \doteq \langle 244, 333 \rangle$ pounds •

[Magnitude of the combined force] $\doteq 413$ pounds • [Angle of inclination] $\doteq \tan^{-1} \left(\frac{333}{244} \right) \doteq 0.94$ radians.

Example 8 Write $\mathbf{z} = \mathbf{u} + 2\mathbf{v} + 3\mathbf{w}$ in the form $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$, where $\mathbf{u} = 3\mathbf{i} - \mathbf{j}$, $\mathbf{v} = \mathbf{j} - 3\mathbf{k}$ and $\mathbf{w} = \mathbf{i} + \mathbf{k}$.

Answer: $\mathbf{z} = 6\mathbf{i} + \mathbf{j} - 3\mathbf{k}$

Example 9 Three adjacent vertices of a parallelogram PQRS in space are $\mathbf{P} = (1, 3, 2)$, $\mathbf{Q} = (4, 5, 3)$, and $\mathbf{R} = (2, -1, 0)$. What are the coordinates of the point \mathbf{S} opposite \mathbf{Q} ?

Answer: Use the schematic sketch in Figure A9. • $\mathbf{S} = (-1, -3, -1)$

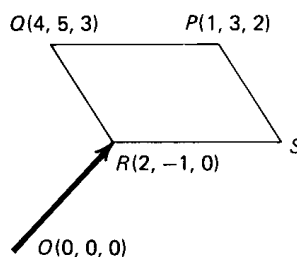


Figure A9

Interactive Examples

Work the following Interactive Examples on Shenk's web page, <http://www.math.ucsd.edu/~ashenk/>:[‡]

Section 12.1: Examples 1, 2, 6, 7

Section 12.3: Examples 1, 2, 6

[‡]The chapter and section numbers on Shenk's web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.