(10/28/08)

Math 10C. Lecture Examples.

Sections 14.1 and 14.2. Partial derivatives†

Example 1 **The table below is from a study of the effect of exercise on the blood pressure of women. $\mathbf{P = P(t, E)}$ is the average blood pressure, measured in millimeters of mercury (mm Hg), of women of age $\mathbf{t}$ years who are exercising at the rate of $\mathbf{E}$ watts.[(1)] (One watt is 0.86 Calories per hour.) What is the approximate rate of change with respect to age of the average blood pressure of forty-five-year old women who are exercising at the rate of 100 watts?**

$\mathbf{P = P(t, E)}$ (millimeters of mercury)

	$t = 25$	$t = 35$	$t = 45$	$t = 55$	$t = 65$
$E = 150$	178	180	197	209	195
$E = 100$	163	165	181	199	200
$E = 50$	145	149	167	177	181
$E = 0$	122	125	132	140	158

Answer: $P_t(45, 100) \approx 1.8$ millimeters of mercury per year (using a right difference quotient); or
$P_t(45, 100) \approx 1.6$ millimeters of mercury per year (using a left difference quotient); or
$P_t(45, 100) \approx 1.7$ millimeters of mercury per year (using a centered difference quotient)

Example 2 **Use the table from Example 1 to find the approximate rate of change with respect to age of the average blood pressure of fifty-five-year-old women who are exercising at the rate of 75 watts.**

Answer: $\left.\frac{\partial P}{\partial E}\right|_{(62,75)} \approx 0.44$ millimeters of mercury per watt

†Lecture notes to accompany Sections 14.1 and 14.2 of *Calculus* by Hughes-Hallett et al.

(1)Data adapted from *Geigy Scientific Tables*, edited by C. Lentner, Vol. 5, Basel, Switzerland: CIBA-GEIGY Limited, 1990, p. 29.

Example 3 **Figure 1 shows level curves of the temperature $\mathbf{T = T(t, h)^\circ F}$ as a function of time $\mathbf{t}$ (hours) and the depth $\mathbf{h}$ (centimeters) beneath the surface of the ground at O'Neil, Nebraska, from noon one day ($t = 0$) until the next morning.[2]**

(a) What was the approximate rate of change of the temperature with respect to time at 4:00 PM at a point 14 centimeters beneath the surface of the ground?

(b) What was the approximate rate of change of the temperature with respect to depth at 4:00 PM at a point 14 centimeters beneath the surface of the ground?

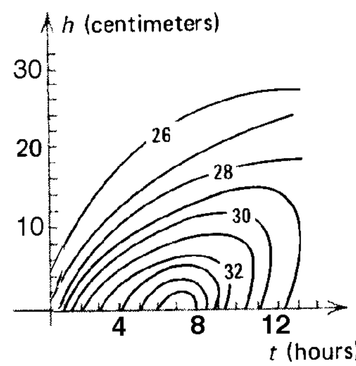


FIGURE 1

Answer: (a) Figure A3 • $T_t(4, 14) \approx 0.5$ degree per hour **(b)** $T_h(4, 14) \approx -0.25$ degree per centimeter

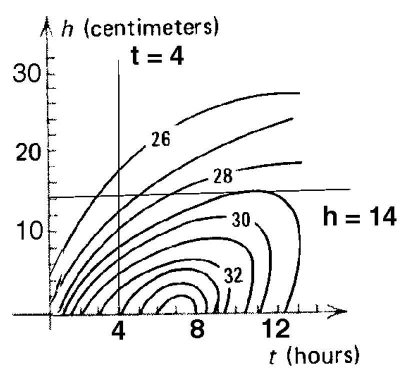


Figure A3

[2] Data adapted from *Fundamentals of Air Pollution* by S. Williamson, Reading, MA: Addison Wesley, 1973.

Example 4 **Find the x- and y-derivatives of $\mathbf{f(x,y) = x^3y - x^2y^5 + x}$.**

Answer: $\dfrac{\partial f}{\partial x} = 3x^2y - 2xy^5 + 1$ • $\dfrac{\partial f}{\partial y} = x^3 - 5x^2y^4$

Example 5 **What are $\mathbf{g_x(2,5)}$ and $\mathbf{g_y(2,5)}$ for $\mathbf{g(x,y) = x^2e^{3y}}$?**

Answer: $g_x(2,5) = 4e^{15}$ • $g_y(2,5) = 12e^{15}$

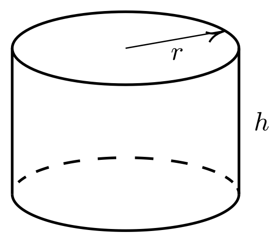
Example 6 **The volume of a right circular cylinder of radius r and height h is equal to the product $\mathbf{V(r,h) = \pi r^2 h}$ of its height h and the area $\mathbf{\pi r^2}$ of its base (Figure 2). What are (a) the rate of change of the volume with respect to the radius and (b) the rate of change of the volume with respect to the height and what are their geometric significance?**

[Area of base] $= \pi r^2$

[Volume] $= \pi r^2 h$

[Lateral surface area] $= 2\pi rh$

FIGURE 2



Answer: **(a)** $\dfrac{\partial V}{\partial r} = 2\pi rh$ is the area of the lateral surface (the sides) of the cylinder.

(b) $\dfrac{\partial V}{\partial h} = \pi r^2$ is the area of the base.

Example 7 **What are the first-order partial derivatives of $\mathbf{f = \ln(xy)}$?**

Answer: $f_x = \dfrac{1}{x}$ • $f_y = \dfrac{1}{y}$

Example 8 **What are (a) $\mathbf{h_{xy}}$ and (b) $\mathbf{h_{yx}}$ for $\mathbf{h(x,y) = \cos x + \sin y + 100xy}$?**

Answer: **(a)** $h_{xy} = 100$ **(b)** $h_{yx} = 100$

Interactive Examples

Work the following Interactive Examples on Shenk's web page, http//www.math.ucsd.edu/˜ashenk/:[‡]

Section 14.3: Examples 1 through 5

Section 14.7: Example 2

Section 14.8: Example 2

[‡]The chapter and section numbers on Shenk's web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.