Math 20A (Shenk). Exam 1. July 13, 2011.
(10 points per problem)
Name
Work alone and use no books, notes, or calculators. Show your work with your answers on 8.5 " $\times 11$ " paper and staple the pages to the exam when you turn them in.
Problem 1 A function $f$ is defined by $f(x)=\left\{\begin{array}{rll}\frac{8}{x^{2}} & \text { for } & 0<x<2 \\ x & \text { for } & 2 \leq x \leq 6 .\end{array}\right.$
What are (a) $\lim _{x \rightarrow 2^{-}} f(x)$, (b) $\lim _{x \rightarrow 2^{+}} f(x)$, and (c) $\lim _{x \rightarrow 2} f(x)$ ? (d) What are the largest intervals on which $f$ is continuous?
Problem 2 Use the definition to find the derivative of $y=\frac{1}{x^{2}+1}$ at $x=0$.
Problem 3 The following table gives the velocity of sound in water $v=v(T)$ (feet per second) as a function of the temperature $T$ (degrees Fahrenheit) of the water. Use this data to find the approximate rate of change of the velocity with respect to the temperature when the temperature is $280^{\circ} \mathrm{F}$.

| $T^{\circ} \mathrm{F}$ | 200 | 240 | 280 | 320 | 360 | 400 | 440 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $v(T)$ (feet per second) | 5079 | 5000 | 4879 | 4724 | 4537 | 4331 | 4081 |

Problem 4 Give an equation of the tangent line to $y=\left(1+x-x^{4}\right)\left(1-x+x^{3}\right)$ at $x=1$.
Problem 5 What is the derivative of $y=\sqrt{x^{2}+2 x+1}$ at $x=5$ ?
Problem 6 Figure 1 shows the graph of a cow's weight as a function of her age $t$. Find the approximate average rate of change of her weight with respect to time from age two to the time when she weighed the most. Then draw the corresponding secant line on the graph.


FIGURE 1


FIGURE 2

Problem 7 The graph of the volume $V=V(h)$ (thousand gallons) of the water in a lake as a function of the depth $h$ (feet) of the water is given above. What is the approximate rate of change of the volume with respect to time when the water is 30 feet deep if it is rising at the rate of 3 feet per day at that time?
Scores:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | Total |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
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