Math 20A, Homework 3, Part 3

Exercise 1 Find the intervals on which **(b)** $y = 1/x^2$ is increasing and decreasing and the open intervals on which its graph is concave up and concave down.

Answer: Defined and continuous on $(-\infty, 0)$ and $(0, \infty)$ • Increasing on $(0, \infty)$ • Decreasing on $(-\infty, 0)$ • Concave up on $(-\infty, 0)$ and $(0, \infty)$

Exercise 2 Use the formulas for $y = x^4 - 12x$ and its first and second derivatives to sketch its graph. Find the intervals on which it is increasing and decreasing and the open intervals on which its graph is concave up and concave down. Find any local or global maxima and minima and inflection points.

Answer: Defined and continuous for all x • $y \to \infty$ as $x \to \pm\infty$ • Decreasing on $(-\infty, \sqrt[3]{3}]$ • Increasing on $[\sqrt[3]{3}, \infty)$ • Global minimum of $y(\sqrt[3]{3}) = -9\sqrt[3]{3} \doteq -13.0$ at $x = \sqrt[3]{3} \doteq 1.44$. • Concave up on $(-\infty, \infty)$ • $y(0) = 0$ • Figure 1

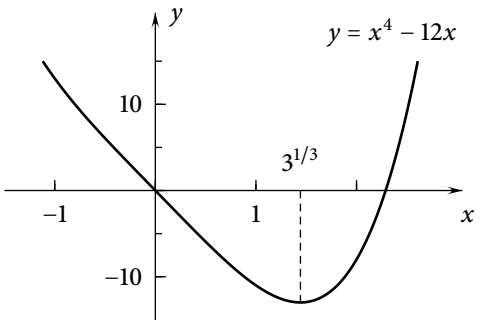

FIGURE 1

Exercise 3 Use the formulas for $y = 3x^2 - x^{-2} + 5$ and its first and second derivatives to sketch its graph. Find the intervals on which it is increasing and decreasing and the open intervals on which its graph is concave up and concave down. Find any local or global maxima and minima and inflection points.

Answer: Defined for $x \neq 0$ • $y \to -\infty$ as $x \to 0$ • $y \to \infty$ as $x \to \pm\infty$ • Decreasing on $(-\infty, 0)$ • Increasing on $(0, \infty)$ • Concave up for $x < -1$ and for $x > 1$ • Concave down for $-1 < x < 0$ and for $0 < x < 1$ • Inflection points: $(-1, 7)$ and $(1, 7)$ • $y(\pm 2) = 16.25$ • Figure 3

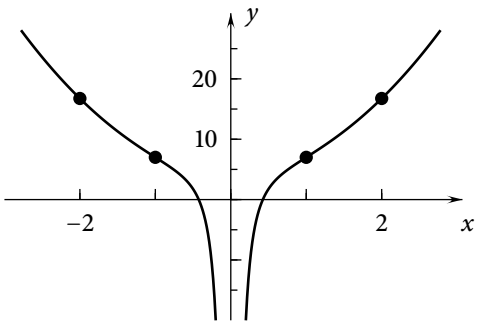

FIGURE 3

Exercise 4 Use the formulas for $f(x) = 2x^{1/2} - x$ and its first and second derivatives to sketch its graph. Find the intervals on which it is increasing and decreasing and the open intervals on which its graph is concave up and concave down. Find any local or global maxima and minima and inflection points.

Answer: Defined and continuous on $[0, \infty)$ • $y \to -\infty$ as $x \to \infty$ • Increasing on $[0,1]$ • Decreasing on $[1, \infty)$ • [Global maximum] = 1 at $x = 1$ • Concave down for $x > 0$ • $y(0) = 0$ • $y(4) = 0$ • Figure 4

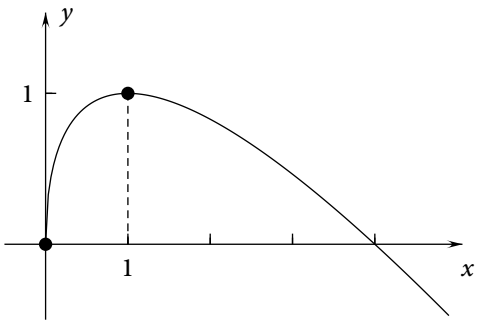

FIGURE 4

Exercise 5 A function $y = h(x)$ is defined and continuous on $[-2, 8]$. The graph of its derivative is shown in Figure 5. **(a)** On what intervals is the function h increasing and decreasing? **(b)** At what values of x does h have local maxima and minima? **(c)** What are the open intervals on which the graph of h is concave up and concave down? **(d)** At what values of x does the graph have inflection points?

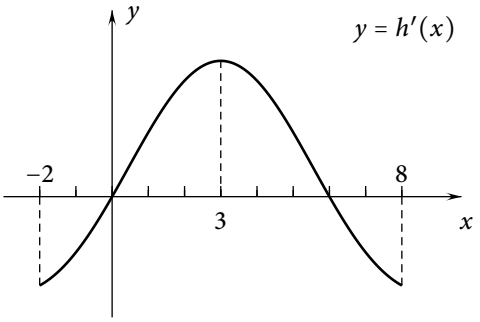

FIGURE 5

Answer: **(a)** Decreasing on $[-2, 0]$ and $[6, 8$ • Increasing on $[0, 6]$ **(b)** Local minimum at $x = 0$ • Local maximum at $x = 6$ **(c)** Concave up on $(-2, 3)$ • Concave down on $(3, 8)$ **(d)** Inflection point at $x = 3$

Exercise 6 Find the critical points of $y = \dfrac{x^2}{x - 3}$.

Exercise 7 What is the global minimum of $f(x) = e^{2x} - x$?

Exercise 8 Find the inflection points of the curve $y = \sin x$.