Math 20A, Homework 4, Part 2

Rogawski Section 4.7: 1, 5, 9, 11, 15, 43

Additional exercises

Exercise 1 **(a)** Give equations of the tangent lines to $y = \sqrt{x} - 1$ and to $y = x^2 - 1$ at $x = 1$. **(b)** Use approximations by the tangent lines in part (a) to predict $\lim_{x \to 1} \dfrac{\sqrt{x} - 1}{x^2 - 1}$. **(c)** Use l'Hopital's Rule to find the limit.

Answer: **(a)** Tangent line to $y = x^{1/2} - 1$ at $x = 1$: $y = \frac{1}{2}(x-1)$ • Tangent line to $y = x^2 - 1$ at $x = 1$: $y = 2(x-1)$

(b) Prediction: $\dfrac{x^{1/2} - 1}{x^2 - 1} \approx \dfrac{\frac{1}{2}(x-1)}{2(x-1)} = \frac{1}{4}$ near $x = 1$ • Prediction: $\lim_{x \to 1} \dfrac{x^{1/2} - 1}{x^2 - 1} = \frac{1}{4}$

(c) $\lim_{x \to 1} \dfrac{x^{1/2} - 1}{x^2 - 1} = \lim_{x \to 1} \dfrac{\frac{1}{2}x^{-1/2}}{2x} = \dfrac{\frac{1}{2}}{2} = \frac{1}{4}$

Exercise 2 Find $\lim_{x \to 0} \dfrac{x^3 + 6x^2 - x}{x^4 + 3x}$ **(a)** by factoring the numerator and denominator and **(b)** by using l'Hopital's Rule.

Answer: **(a)** For $x \neq 0$, $\dfrac{x^3 + 6x^2 - x}{x^4 + 3x} = \dfrac{x(x^2 + 6x - 1)}{x(x^3 + 3)} = \dfrac{x^2 + 6x - 1}{x^3 + 3}$ •

$\lim_{x \to 0} \dfrac{x^3 + 6x^2 - x}{x^4 + 3x} = \lim_{x \to 0} \dfrac{x^2 + 6x - 1}{x^3 + 3} = -\frac{1}{3}$

(b) $\lim_{x \to 0} \dfrac{x^3 + 6x^2 - x}{x^4 + 3x} = \lim_{x \to 0} \dfrac{\frac{d}{dx}(x^3 + 6x^2 - x)}{\frac{d}{dx}(x^4 + 3x)} = \lim_{x \to 0} \dfrac{3x^2 + 6x - 1}{4x^3 + 3} = -\frac{1}{3}$

Exercise 3 Find $\lim_{x \to \infty} \dfrac{e^x + 5}{e^{-x} + 10}$.

Answer: $\lim_{x \to \infty} \dfrac{e^x + 5}{e^{-x} + 10} = \infty$ (l'Hopital's Rule is not needed.)

Exercise 4 Use l'Hopital's Rule and the First-Derivative Test to draw the curve $y = x^2 e^{-x/2}$. Use the window $-2 \le x \le 10$, $-1 \le y \le 5$.

Answer: Properties: $y(x)$ is defined and continuous for all x. • $y \to \infty$ as $x \to -\infty$ • $y \to 0$ as $x \to \infty$ by L'Hopital's Rule • $y(x)$ is decreasing on $(-\infty, 0]$ and $[4, \infty)$ and increasing on $[0, 4]$. • Global minimum of 0 at $x = 1$ • Local maximum of $16e^{-2}$ at $x = 4$ • Figure 1

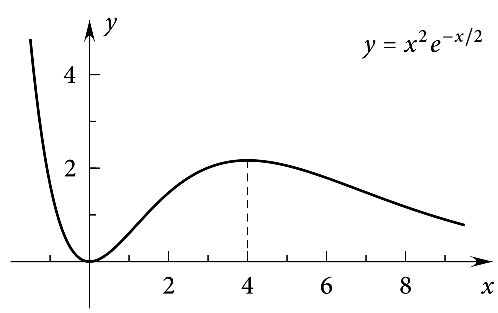

FIGURE 1

Exercise 5 Use l'Hopital's Rule and the First-Derivative Test to draw the graph of $y = \dfrac{\ln x}{x}$. Use the window $-2 \le x \le 10$, $-1 \le y \le 0.75$.

Answer: Properties: $y(x)$ is defined and continuous on $(0, \infty)$. • $y \to -\infty$ as $x \to 0^+$ • $y(x) \to 0$ as $x \to \infty$ by l'Hopital's Rule • $y(x)$ is increasing on $(0.e]$ and decreasing on $[e, \infty)$. • Global maximum of $\dfrac{1}{e}$ at $x = e$ • Figure 2

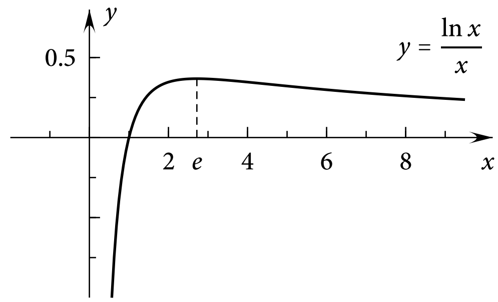

FIGURE 2