Homework 4, Part 3.

Exercise 1   Use the formula for the area of a triangle to find the value of $\int_0^3 (x-1)\, dx$.

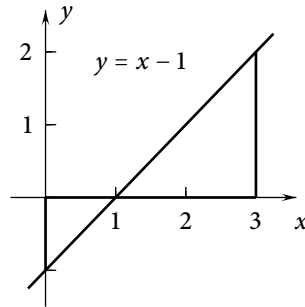Answer: Figure 1 • $\int_0^3 (x-1)\, dx = \frac{3}{2}$

FIGURE 1

Exercise 2   Use a formula from geometry to evaluate $\int_{-4}^0 \sqrt{16 - x^2}\, dx$.

Answer: Figure 2 • $\int_{-4}^0 \sqrt{16 - x^2}\, dx = 4\pi$
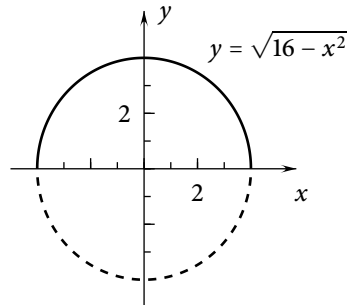

FIGURE 2

Exercise 3   Calculate $\sum_{j=1}^6 j(j-1)(j-2)$.

Answer: $0 + 0 + 3(2)(1) + 4(3)(2) + 5(4)(3) + 6(5)(4) = 210$

Exercise 4   Express $1^2 + 2^2 + 3^2 + \cdots + 99^2$ with summation notation.

Answer: $\sum_{j=1}^{99} j^2$

Exercise 5   What is $\int_{-7}^7 [6f(x) + 3g(x)]\, dx$ if $\int_{-7}^7 f(x)\, dx = 4$ and $\int_{-7}^7 g(x)\, dx = -5$?

Answer: $\int_{-7}^7 [6f(x) + 3g(x)]\, dx = 9$

Exercise 6   What is $\int_1^2 h(x)\, dx$ if $\int_1^6 h(x)\, dx = 10$ and $\int_2^6 h(x)\, dx = 7$?

Answer: $\int_1^2 h(x)\, dx = 3$

Exercise 7   Use areas to evaluate $\int_3^1 7\,dx$.

Answer: $\int_3^1 7\,dx = -14$

Exercise 8   Use the formulas for areas of rectangles and circles to evaluate $\int_0^2 (5 - 3\sqrt{4 - x^2})\,dx$.

Answer: $\int_0^2 (5 - 3\sqrt{4 - x^2})\,dx = 10 - 3\pi$

Exercise 9   Calculate **(a)** the left and **(b)** the right Riemann sums for $\int_0^2 P(x)\,dx$ relative to the partition of $[0, 2]$ into four equal subintervals, where $y = P(x)$ is the function whose graph is shown in Figure 3.
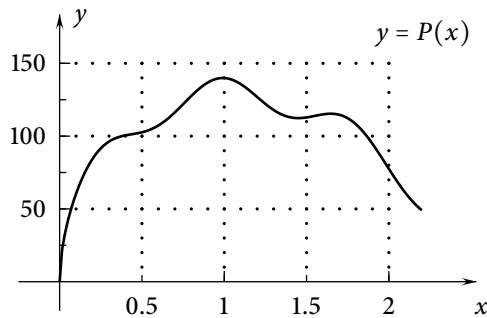
FIGURE 3