Math 20A. Lecture 1.

Read Section 0.1 of my planned e-Calculus book. There is a link to it on the course web page, http://www.math.ucsd.edu/ ashenk/.

Inequalities

Example 1 Solve the inequality $5 - 2x \leq 9$ for x.

Answer: $x \geq -2$

Example 2 Solve $|x^3 - 1| > 2$.

Answer: $x < -1$ or $x > \sqrt[3]{3}$

Translation, magnification, contraction

Example 3 Explain the shape of the graph of the function $y = x^2 - 2x + 3$ in Figure 1 by completing the square.

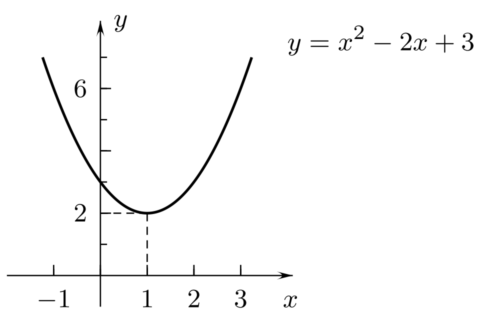


FIGURE 1

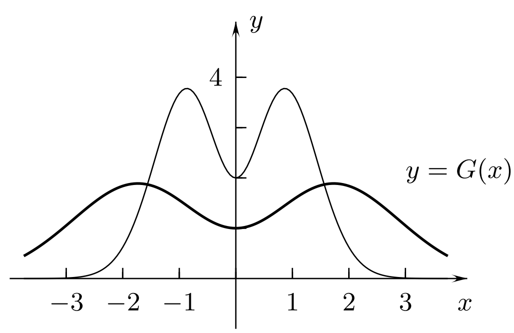


FIGURE 2

Answer: $y = (x-1)^2 + 2$ is $y = x^2$ translated 1 unit to the right and then raised 2 units.

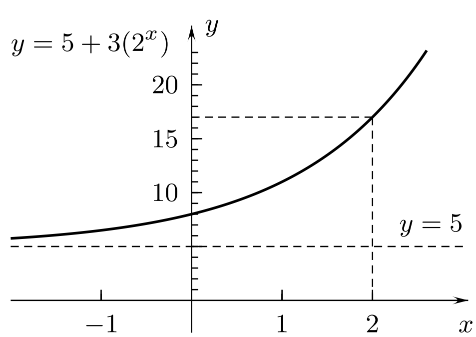
Example 4 The curve drawn with a heavy line in Figure 2 is the graph of $y = G(x)$. Is the other curve the graph of $y = \frac{1}{2}G(2x), y = \frac{1}{2}G(\frac{1}{2}x), y = \frac{1}{2}G(x)$, or $y = 2G(2x)$?

Answer: The curve drawn with a fine line has the equation $y = 2G(2x)$.

Exponential functions

Example 5 Explain the shape of the curve $y = 5 + 3(2^x)$ in Figure 3.

FIGURE 3



Answer: The curve $y = 5 + 3(2^x)$ is $y = 2^x$ magnified vertically by a factor of 3 and then translated up 5 units. • Also $y(0) = 5 + 3(2^0) = 8$, $y(2) = 5 + 3(2^2) = 17$, and $y = 5$ is a horizontal asymptote.

The NATURAL EXPONENTIAL FUNCTION $y = e^x$ is defined by

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \quad \text{or} \quad e^x = \lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n.$$

Its graph and the graph of its inverse, the NATURAL LOGARITHM $y = \ln x$, are in Figure 4. The number e has the decimal value $e = 2.71828183\ldots$.

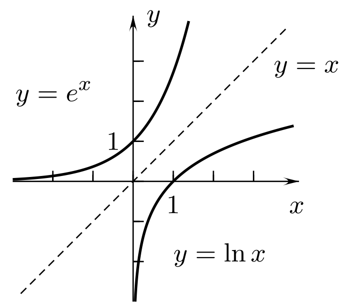


FIGURE 4

Example 6 Solve the equation $e^{3x} = 100$ for x.

Answer: $x = \frac{1}{3}\ln(100)$

Trigonometric functions

In calculus, angles are measured in radians. To find the radian measure of a positive (counterclockwise) angle x, put its vertex at the center of a circle of radius 1 as in Figure 5. The radian measure of the angle is the length of the arc it subtends (sweeps out) on the circle as it is generated (including all revolutions if $x > 2\pi$). The radian measure of a negative (clockwise) angle is the negative of the length of the arc it subtends.

Recall the definitions of $\cos x, \sin x$, and $\tan x$ from trigonometry: If x is a positive acute angle ($0 < x < \frac{1}{2}\pi$) in a right triangle then $\cos x$ equals the length of the adjacent side divided by the length of the hypotenuse, $\sin x$ equals the length of the opposite side divided by the length of the hypotenuse, and $\tan x$ equals the length of the opposite side divided by the length of the adjacent side.

To find $\cos x$ and $\sin x$ for a general angle x, let (u, v) be the coordinates in a uv-plane of the point a positive distance r from the origin on the terminal side of the angle, as in Figure 6. Then

$$\cos x = \frac{u}{r}, \qquad \sin x = \frac{v}{r}, \qquad \tan x = \frac{v}{u}.$$

(The tangent is not defined if the point is on the v-axis and $u = 0$.) Notice that this is the definition from trigonometry if x is a positive acute angle.

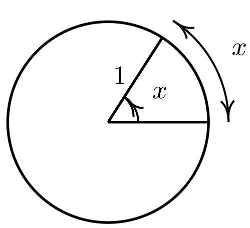


FIGURE 5

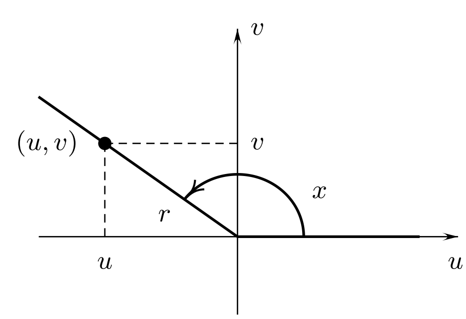


FIGURE 6

Choosing $r = 1$ in this definition gives a way to relate $\cos x$ and $\sin x$ to the unit circle (the circle of radius 1 with center at the origin) in the uv-plane (Figure 7): The intersection of the terminal side of the angle with the circle has coordinates $(\cos x, \sin x)$.

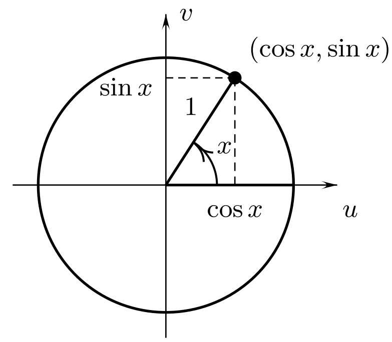


FIGURE 7

This formulation of the definition can be used to show that the graphs $y = \cos x$ and $y = \sin x$ have the shapes in Figures 8 and 9.

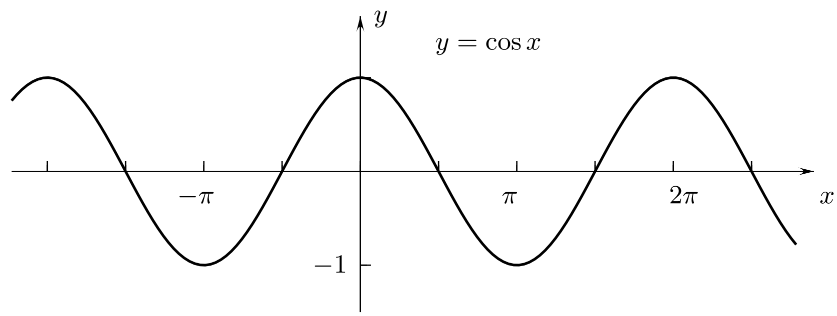


FIGURE 8

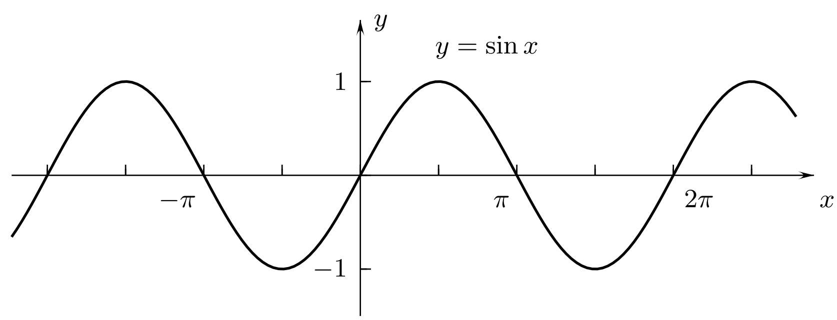


FIGURE 9

The tangent function can be defined in terms of the sine and cosine by the formula

$$\tan x = \frac{\sin x}{\cos x}$$

for all x such that $\cos x \neq 0$. It also has the following geometric definition using the unit circle.

Draw the terminal side of the angle x and the vertical line $u = 1$ at the right of the unit circle as in Figures 10, 11, and 12. The vertical, v-coordinate of the intersection of these lines is $\tan x$ because the horizontal coordinate is $u = 1$ so that $\tan x = v/u = v$. (The function $y = \tan x$ is called the "tangent" function because the line $u = 1$ in Figures 10, 11, and 12 is tangent to the unit circle.)

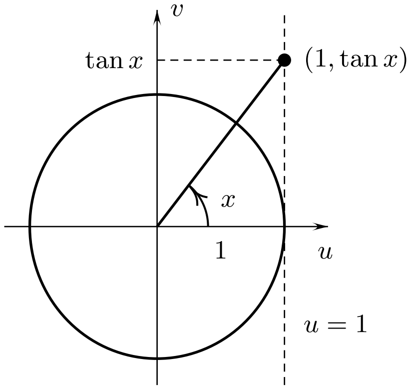


FIGURE 10

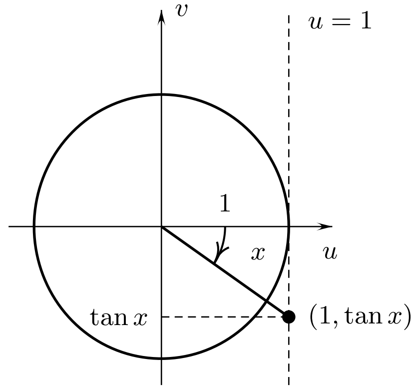


FIGURE 11

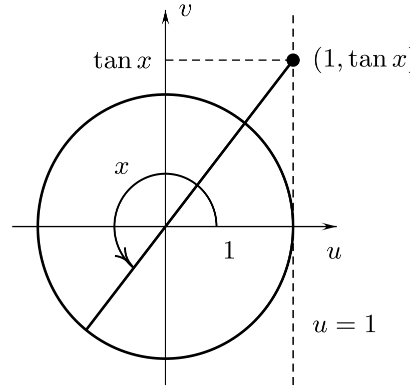
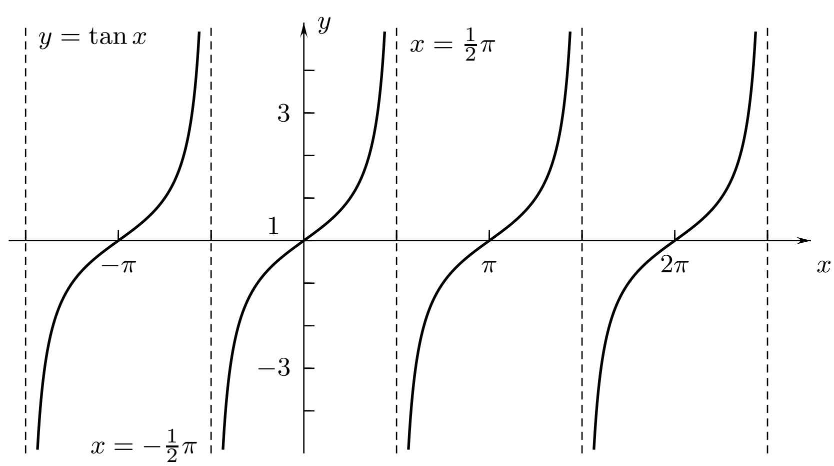


FIGURE 12

This definition can be used to show that the graph $y = \tan x$ has the shape in Figure 13. Notice that $\tan x$ has period π, while $\cos x$ and $\sin x$ have period 2π.

FIGURE 13



Interactive Examples on the course web page

Section 0.1: 1–5

Section 0.2: 1–5

Section 0.3: 1–8