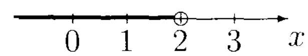
Quiz 1 Solutions

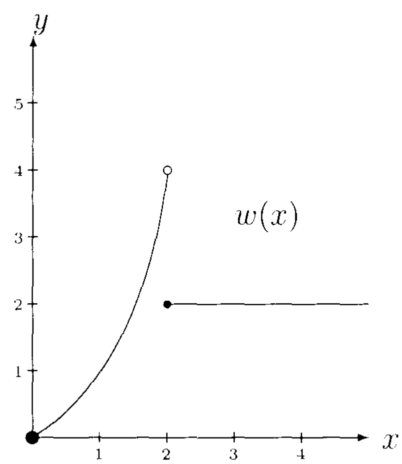
1)

$$\begin{aligned} 2x+1 &> 5x-5 \\ 2x+1+5 &> 5x-5+5 \\ 2x+6 &> 5x \\ 2x+6-2x &> 5x-2x \\ 6 &> 3x \\ 2 &> x \end{aligned}$$

The graph of the solution set $\{x : x < 2\}$ is:



2) (a)



(b) $\lim_{x\to 0^+} w(x) = 0$ (c) $\lim_{x\to 1} w(x) = 1$ (d) $\lim_{x\to 2^-} w(x) = 4$ (e) $\lim_{x\to 3} w(x) = 2$

(f) If $x \geq 2$ then $w(x) = 2$ so in order to solve $w(x) = 3$ we can assume $0 \leq x < 2$. If $0 \leq x < 2$ then $w(x) = x^2$ and so we must solve the equation $x^2 = 3$.

$$\begin{aligned} x^2 &= 3 \\ x &= \pm\sqrt{3} \end{aligned}$$

Since $0 \leq x < 2$, we can ignore the solution $x = -\sqrt{3}$. The solution to $w(x) = 3$ is $x = \sqrt{3}$.

(g) w is continuous on the intervals $[0, 2)$ and $[2, \infty)$.

3) By definition, $f'(a) = \lim_{x \to a} \dfrac{f(x) - f(a)}{x - a}$. Applying this definition with $f(x) = \frac{4}{x+2}$ and $a = 2$:

$$f'(2) = \lim_{x \to 2} \frac{\frac{4}{x+2} - \frac{4}{2+2}}{x - 2} = \lim_{x \to 2} \frac{\frac{4}{x+2} - 1}{x - 2} = \lim_{x \to 2} \frac{\frac{4}{x+2} - \frac{x+2}{x+2}}{x - 2} = \lim_{x \to 2} \frac{\frac{-(x-2)}{x+2}}{x - 2} = \lim_{x \to 2} \frac{-1}{x+2}.$$

Since $y = \frac{-1}{x+2}$ is continuous at $x = 2$,

$$\lim_{x \to 2} \frac{-1}{x+2} = \frac{-1}{2+2} = -\frac{1}{4}.$$

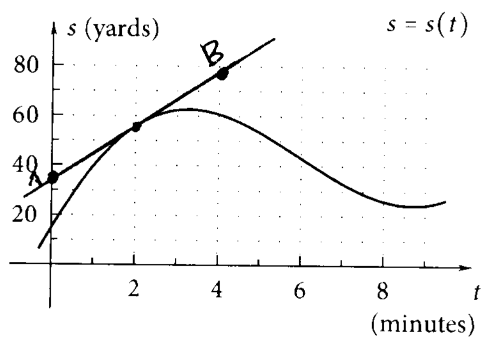
We conclude $f'(2) = -\frac{1}{4}$.

One could also compute $\lim_{h \to 0} \dfrac{\frac{4}{(h+2)} - \frac{4}{2+2}}{h}$ in order to determine $f'(2)$.

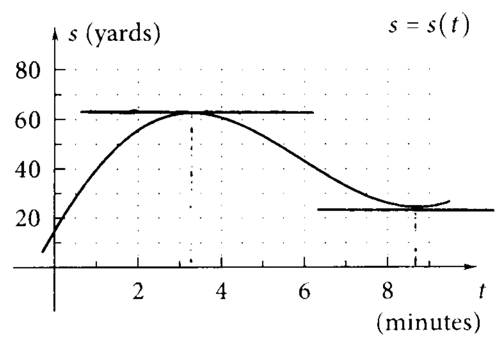
4) (a) We start by drawing an approximation to the tangent line to $s = s(t)$ at $t = 2$ (see Figure I below). Next choose two points on the line A and B. The coordinates of A are (0.35) and the coordinates of B are $(4, 75)$. The slope of the line through points A and B is

$$\frac{75 - 35}{4 - 0} = \frac{40}{4} = 10.$$

The object's approximate velocity in the positive s-direction at $t = 2$ is 10 yards per minute.



(b) The velocity is 0 when the graph of $s = s(t)$ has a horizontal tangent line. Looking at the graph of $s(t)$, this occurs when $t \approx 3.25$ minutes and $t \approx 8.75$ minutes (see Figure II below).



5) The equation of the tangent line at $x = 7$ is $y = y(7) + y'(7)(x - 7)$. We need to compute $y(7)$ and $y'(7)$.

$$y(7) = \frac{7}{7+3} = \frac{7}{10}.$$

To compute $y'(7)$ we need to find $y' = \frac{d}{dx}\left(\frac{x}{x+3}\right)$. To compute y' we use the Quotient Rule.

$$y' = \frac{d}{dx}\left(\frac{x}{x+3}\right) = \frac{(x+3)(x)' - (x)(x+3)'}{(x+3)^2} = \frac{(x+3)(1) - (x)(1)}{(x+3)^2} = \frac{3}{(x+3)^2}.$$

Therefore $y'(7) = \frac{3}{(7+3)^2} = \frac{3}{100}$. The equation of the tangent line at $x = 7$ is

$$y = \frac{7}{10} + \frac{3}{100}(x - 7).$$

6) Let $V(t)$ be the amount of oil (in barrels) at time t (in days). Let $p(t)$ be the price per barrel of oil (in dollars) at time t. Finally, let $F(t)$ be the value (in dollars) at time t. Then $F(t) = V(t)p(t)$ dollars. By the Product Rule,

$$F'(t) = p(t)V'(t) + V(t)p'(t). \tag{1}$$

We are given $p(0) = 95$, $p'(0) = -5$, $V(0) = 2000$, and $V'(0) = 100$. Substituting into Equation (1),

$$F'(0) = (95)(100) + (2000)(-5) = -500.$$

The value of our oil is decreasing at a rate of 500 dollars per day.