

Math 20B. Lecture Examples.

Sections 5.1–5.5. The definite integral, review[†]

Example 1 Use the formula for the area of a triangle to evaluate $\int_{-3}^3 (x+1) dx$.

Answer: Figure A1 • $\int_{-3}^3 (x+1) dx = 6$

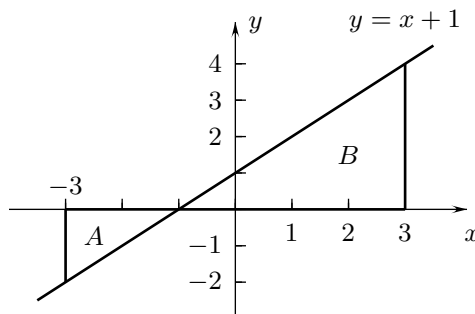


Figure A1

Example 2 Use the graphs of $y = F(x)$ and its derivative $r = F'(x)$ to show that the Fundamental Theorem,

$$\int_a^b F'(x) dx = F(b) - F(a)$$

holds with $a = 0$ and $b = 3$ for $F(x) = \begin{cases} 3x + 2 & \text{for } 0 \leq x \leq 1 \\ 6 - x & \text{for } 1 < x \leq 3. \end{cases}$

Answer: The graphs of F and its derivatives are shown in Figure A2a and A2b. • $\int_0^3 F'(x) dx = 1$ • $F(3) - F(0) = 1$

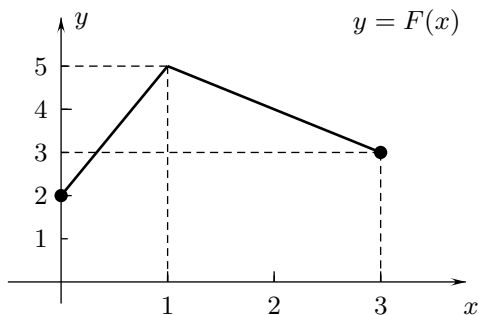


Figure A2a

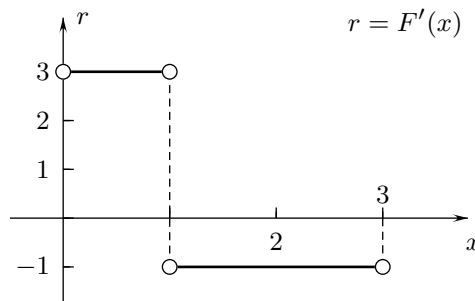


Figure A2b

[†]Lecture notes to accompany Sections 5.1–5.5 of *Calculus, Early Transcendentals* by Rogawski

Example 3 Figure 6 shows the graph of the continuous derivative $r = G'(x)$ of a continuous function $y = G(x)$. Region A in the drawing has area 89 and region B has area 62. What is $G(6) - G(1)$?

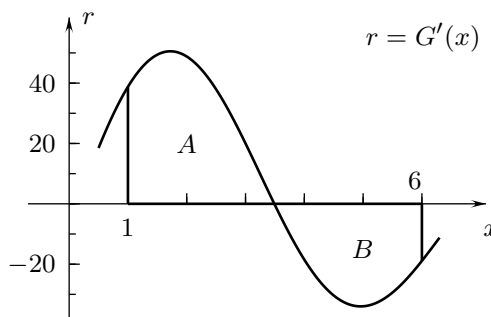


FIGURE 6

Answer: $G(6) - G(1) = 27$

Example 4 The St. Francis dam, constructed in 1928 northeast of the present Magic Mountain near Los Angeles, was designed by William Mulholland, the self-educated engineer who had been the chief architect of the Los Angeles–Owens River Aqueduct that supplies much of the water to Los Angeles. When the reservoir was filled for the first time, the dam broke, flooding the San Francisquito and Santa Clara River valleys and drowning 450 people. Figure 7 shows the graph of the rate of flow of water $r = r(t)$ from the St. Francis dam in a 90-minute period starting 15 minutes before it broke.⁽¹⁾ Estimate the total volume of water that flowed from the dam for $0 \leq t \leq 90$.

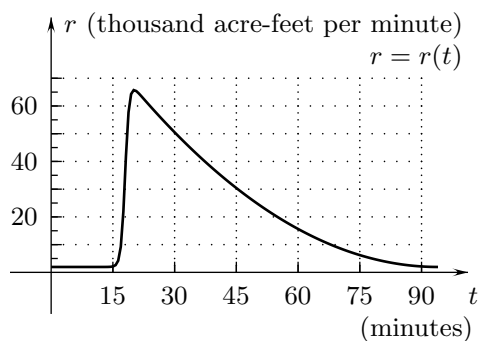


FIGURE 7

Answer: [Total volume] = $\int_0^{90} r(t) dt$ • One answer: Figure A4 • [Total volume] ≈ 1950 thousand acre-feet

⁽¹⁾Data adapted from “A man, a dam and a disaster: Mulholland and the St. Francis Dam” by J. Rogers, *Ventura County Historical Society Quarterly*, Vol. 77, Ventura California: Ventura County Historical Society, 1995, p. 76.

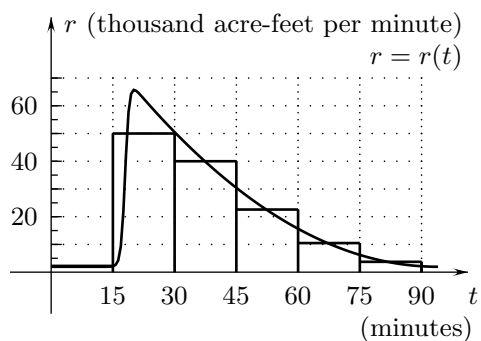


Figure A4

Basic integration formulas

Basic differentiation formulas translate into the following integration formulas:

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C \text{ for } n \neq -1$$

$$\int \frac{1}{x} dx = \ln |x| + C$$

$$\int e^x dx = e^x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

$$\int \cosh x dx = \sinh x + C$$

$$\int \sinh x dx = \cosh x + C$$

Example 5 Find the area of the region bounded by the curve $y = 3x^2 - x^3$ and the x -axis.

Answer: Figure A5 • [Area] = $\frac{27}{4}$

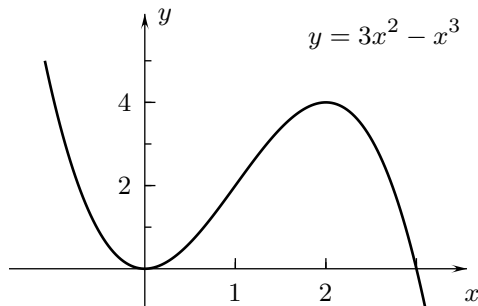


Figure A5

Example 6 Find a formula for the function $y = g(x)$ such that $g'(x) = e^x$ for all x and $g(2) = 10$.

Answer: $g(x) = e^x + 10 - e^2$

Example 7 What is the area of the region between $y = \sec^2 x$ and the x -axis for $-1 \leq x \leq 1$?

Answer: Figure A7 • [Area] = $2 \tan(1) \doteq 3.1148$

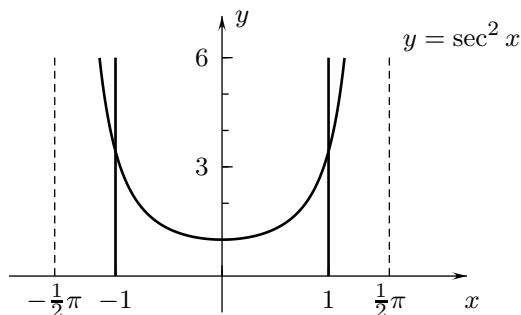


Figure A7

Example 8 A car is 30 miles north of a town at time $t = 0$ (hours) and its velocity toward the north is $v(t) = 60 + 5 \cos t + 8 \sin t$ miles per hour for $0 \leq t \leq 3$. Where is it at $t = 3$?

Answer: The car is $5 \sin(3) - 8 \cos(3) + 218 \doteq 226.63$ miles north of the town at $t = 3$.

Example 9 The table below lists the rate $r = r(t)$ at which residents of the U.S. spent money on commodities and services, as measured on January 1 every other year just before and during the Great Depression.⁽²⁾ (a) Express the total spent from the beginning of 1929 to the beginning of 1939 as an integral. (b) Estimate the integral by using the left Riemann sum corresponding to the partition $1929 < 1931 < 1933 < 1935 < 1937 < 1939$.

RATE OF SPENDING (BILLION DOLLARS PER YEAR)

t	1929	1931	1933	1935	1937	1939
$r(t)$	77.2	60.5	45.8	55.7	66.5	72.0

Answer: (a) [Total spent from the beginning of 1929 to the beginning of 1939] = $\int_{1929}^{1939} r(t) dt$

(b) Figure A9 • [Left Riemann sum] = 611.4 • Approximately 611.4 billion dollars was spent.

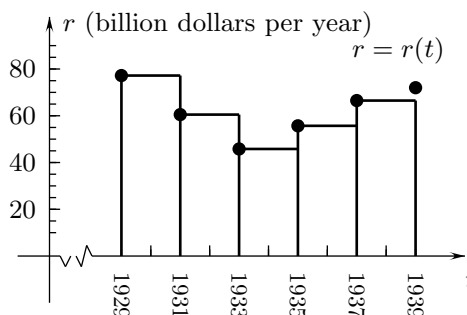


Figure A9

Example 10 A ball is thrown straight up in the air from six feet above the ground at time $t = 0$ (seconds) has upward velocity $v = 64 - 32t$ (feet per second) at time t for $0 \leq t \leq 4$. (a) How high is the ball at $t = 4$? (b) How far does the ball travel for $0 \leq t \leq 4$?

Answer: (a) Figure A10 • The ball is six feet above the ground at $t = 4$. (b) The ball travels 128 feet.

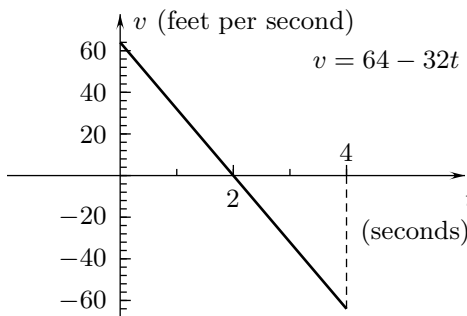


Figure A10

⁽²⁾Data adapted from *Historical Statistics of the United States*, Washington, D.C.: U.S. Department of Commerce, p. 319.

Example 11 Imagine you had \$50,000 in investments at the beginning of the year 2001 and that their value was increasing at the rate of $R = 800/t$ dollars per year t years after the beginning of 2000 for $t \geq 1$. What would have been the value of your investments at the beginning of 2003?

Answer: At the beginning of 2003, the investments would have been worth $800 \ln(3) + 50,000 \doteq \$50,878.89$.

Interactive Examples

Work the following Interactive Examples on Shenk's web page, <http://www.math.ucsd.edu/~ashenk/>:[‡]

Section 6.3: Example 4

Section 6.5: Examples 1–4

Section 6.7: Examples 1–5, 8, 9

[‡]The chapter and section numbers on Shenk's web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.