

Math 20C. Lecture Examples.

Section 12.2. Vectors in three dimensions[†]

Example 1 Sketch the box consisting of all points (x, y, z) with $0 \leq x \leq 2$, $2 \leq y \leq 3$, and $0 \leq z \leq 2$. What are the coordinates of the eight corners of the box?

Answer: Figure A1. • The corners of its base, ordered counterclockwise, are $(2, 2, 0)$, $(2, 3, 0)$, $(0, 3, 0)$, and $(0, 2, 0)$. • The corners of its top are $(2, 2, 2)$, $(2, 3, 2)$, $(0, 3, 2)$, and $(0, 2, 2)$.

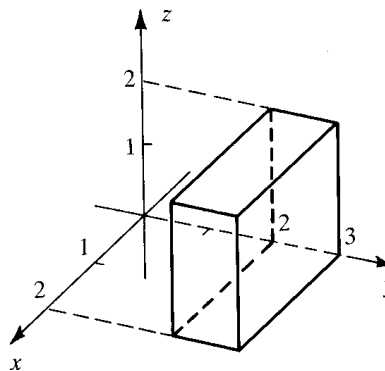


Figure A1

Example 2 What is the length of the diagonals of the box from Example 1?

Answer: The length of each of its four diagonals is $\sqrt{1^2 + 2^2 + 2^2} = 3$

Example 3 Describe the set of points defined by the equation,
 $(x - 1)^2 + (y - 2)^2 + (z - 3)^2 = 16$.

Answer: $(x - 1)^2 + (y - 2)^2 + (z - 3)^2 = 16$ is the sphere of radius 4 with its center at $(1, 2, 3)$.

Example 4 Describe the set of points in xyz -space defined by the equation,
 $x^2 + y^2 = 25$.

Answer: $x^2 + y^2 = 25$ is the cylinder of radius 5 with the z -axis as its axis.

Example 5 Write $\mathbf{z} = \mathbf{u} + 2\mathbf{v} + 3\mathbf{w}$ in the form $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$, where $\mathbf{u} = 3\mathbf{i} - \mathbf{j}$, $\mathbf{v} = \mathbf{j} - 3\mathbf{k}$ and $\mathbf{w} = \mathbf{i} + \mathbf{k}$.

Answer: $\mathbf{z} = 6\mathbf{i} + \mathbf{j} - 3\mathbf{k}$

Example 6 Three adjacent vertices of a parallelogram PQRS in space are $\mathbf{P} = (1, 3, 2)$, $\mathbf{Q} = (4, 5, 3)$, and $\mathbf{R} = (2, -1, 0)$. What are the coordinates of the point S opposite Q?

Answer: Use the schematic sketch in Figure A6. • $\mathbf{S} = (-1, -3, -1)$

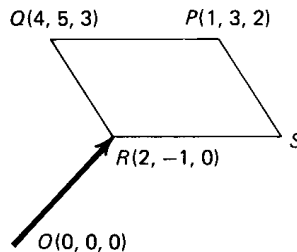


Figure A6

[†]Lecture notes to accompany Section 12.2 of *Calculus, Early Transcendentals* by Rogawski.

Example 7 Give parametric equations for the line L through the point $(6,4,3)$ and parallel to the vector $2\mathbf{i} + 5\mathbf{j} - 7\mathbf{k}$.

Answer: $L: x = 6 + 2t, y = 4 + 5t, z = 3 - 7t$

Example 8 Give parametric equations for the line L through $P = (5, 3, 1)$ and $Q = (7, -2, 0)$.

Answer: $L: x = 5 + 2t, y = 3 - 5t, z = 1 - t$

Example 9 Find the intersection of the lines $L_1: x = 2 - t, y = 3 + t, z = 4 - 2t$ and $L_2: x = -3 + t, y = -1 + 2t, z = 9 - 3t$

Answer: Intersection: $(0, 5, 0)$

Example 10 A bucket of water is supported by two ropes fastened at the same point on its handle. The forces by the ropes on the bucket, relative to xyz -space with an upward pointing z -axis, are $\mathbf{F}_1 = \langle 3, a, 6 \rangle$ (pounds) and $\mathbf{F}_2 = \langle b, -4, 5 \rangle$ (pounds). What are the numbers a and b ? How much does the bucket weigh?

Answer: $a = 4, b = -3$ and the bucket weighs 11 pounds.

Interactive Examples

Work the following Interactive Examples on Shenk's web page, <http://www.math.ucsd.edu/~ashenk/>:[‡]

Section 12.3: Examples 1, 2, and 6

Section 12.5: Examples 1 and 2

[‡]The chapter and section numbers on Shenk's web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.