Math 20C. Lecture Examples.

Section 12.3. The dot product and angles between vectors[†]

Example 1 **Calculate $\mathbf{v}\cdot\mathbf{w}$ for $\mathbf{v} = \langle 6, -2\rangle$ and $\mathbf{w} = \langle 4, 3\rangle$.**

Answer: $\mathbf{v}\cdot\mathbf{w} = 18$.

Example 2 **What is $\mathbf{v}\cdot\mathbf{w}$ for $\mathbf{v} = \langle 6, -2, 3\rangle$ and $\mathbf{w} = \langle 4, 3, -6\rangle$?**

Answer: $\mathbf{v}\cdot\mathbf{w} = 0$

Example 3 **Find an angle θ between the vectors $\mathbf{v} = \langle 4, 1\rangle$ and $\mathbf{w} = \langle 2, 4\rangle$ in Figure 1. Give exact and approximate decimal values.**

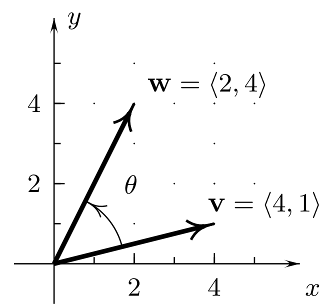

FIGURE 1

Answer: $\theta = \cos^{-1}\left(\dfrac{12}{\sqrt{17}\sqrt{20}}\right) \doteq 0.862$ radians

Example 4 **Find the constant k such that the vectors $\langle -3, -1\rangle$ and $\langle \mathbf{k}, -2\rangle$ are perpendicular. Then draw the two vectors.**

Answer: $k = \frac{2}{3}$ • The vectors are $\langle \frac{2}{3}, -2\rangle$ and $\langle -3, -1\rangle$. • Figure A4

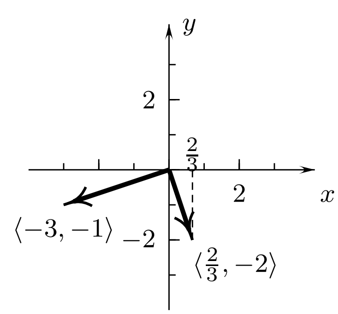

Figure A4

[†]Lecture notes to accompany Section 12.3 of *Calculus, Early Transcendentals* by Rogawski.

Example 5 **Find the component of $\mathbf{u} = \langle -6, 3 \rangle$ along $\mathbf{v} = \langle 2, 2 \rangle$. Give the exact and approximate decimal values.**

Answer: [Component of $\mathbf{u}$ along $\mathbf{v}$] $= -\frac{3}{2}\sqrt{2} \doteq -2.12$ • Figure A5

$$\begin{bmatrix} \text{The component of } \mathbf{u} \\ \text{along } \mathbf{v} \end{bmatrix} = t$$

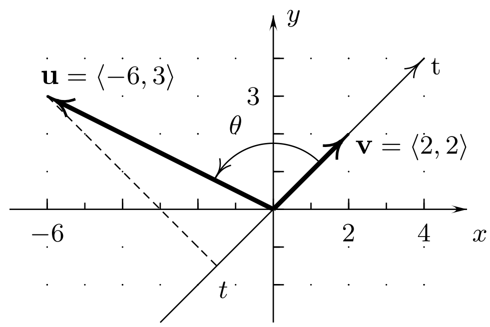

Figure A5

Example 6 **What is the projection of $\mathbf{u} = \langle -1, 3, 4 \rangle$ along $\mathbf{v} = \langle 3, 2, 1 \rangle$?**

Answer: $\text{proj}_{\mathbf{v}}(\mathbf{u}) = \langle \frac{3}{2}, 1, \frac{1}{2} \rangle$

Interactive Examples

Work the following Interactive Examples on Shenk's web page, http//www.math.ucsd.edu/˜ashenk/:[‡]

Section 12.3: Examples 1–5

Section 12.4: Examples 3–5

[‡]The chapter and section numbers on Shenk's web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.