Math 20C. Lecture Examples.

Section 12.3. The dot product and angles between vectors†

Example 1 Calculate \( v \cdot w \) for \( v = \langle 6, -2 \rangle \) and \( w = \langle 4, 3 \rangle \).
Answer: \( v \cdot w = 18 \).

Example 2 What is \( v \cdot w \) for \( v = \langle 6, -2, 3 \rangle \) and \( w = \langle 4, 3, -6 \rangle \)?
Answer: \( v \cdot w = 0 \)

Example 3 Find an angle \( \theta \) between the vectors \( v = \langle 4, 1 \rangle \) and \( w = \langle 2, 4 \rangle \) in Figure 1. Give exact and approximate decimal values.

Answer: \( \theta = \cos^{-1} \left( \frac{12}{\sqrt{17} \sqrt{20}} \right) \approx 0.862 \text{ radians} \)

Example 4 Find the constant \( k \) such that the vectors \( \langle -3, -1 \rangle \) and \( \langle k, -2 \rangle \) are perpendicular. Then draw the two vectors.
Answer: \( k = \frac{2}{3} \) • The vectors are \( \langle \frac{2}{3}, -2 \rangle \) and \( \langle -3, -1 \rangle \). • Figure A4

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†Lecture notes to accompany Section 12.3 of Calculus, Early Transcendentals by Rogawski.
Example 5  
Find the component of $u = (-6, 3)$ along $v = (2, 2)$. Give the exact and approximate decimal values.

Answer: $[\text{Component of } u \text{ along } v] = \frac{-3}{2} \sqrt{2} \approx -2.12 \quad \bullet \quad \text{Figure A5}$

Example 6  
What is the projection of $u = (-1, 3, 4)$ along $v = (3, 2, 1)$?

Answer: $\text{proj}_v(u) = (\frac{3}{2}, 1, \frac{1}{2})$

Interactive Examples

Work the following Interactive Examples on Shenk’s web page, http://www.math.ucsd.edu/~ashenk/:

Section 12.3: Examples 1–5
Section 12.4: Examples 3–5

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†The chapter and section numbers on Shenk’s web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.