(10/1/08)

Math 20C. Lecture Examples.

Section 12.4. The cross product[†]

Example 1 **Evaluate the determinant,** $\begin{vmatrix} \mathbf{3} & \mathbf{2} & \mathbf{4} \\ \mathbf{-1} & \mathbf{0} & \mathbf{6} \\ \mathbf{5} & \mathbf{1} & \mathbf{-2} \end{vmatrix}$.

Answer: The given determinant equals 34.

Example 2 **Find the cross product of $\mathbf{v} = \langle 3, 1, -2\rangle$ and $\mathbf{w} = \langle 0, 4, 2\rangle$.**

Answer: $\mathbf{v} \times \mathbf{w} = \langle 10, -6, 12\rangle$

Example 3 **As a partial check of the result of Example 2, show that each the given vectors is perpendicular to the calculated cross product.**

Answer: Let $\mathbf{u} = \langle 10, -6, 12\rangle$ be the calculated cross product. • $\mathbf{v} \cdot \mathbf{u} = 0$ • $\mathbf{w} \cdot \mathbf{u} = 0$

Example 4 **Find a nonzero vector perpendicular to $\mathbf{v} = 4\mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{w} = 2\mathbf{i} - \mathbf{k}$.**

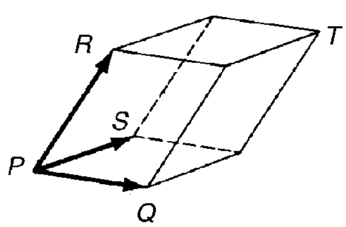
Answer: One answer: The cross product $\mathbf{v} \times \mathbf{w} = \mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$ is perpendicular to $\mathbf{v}$ and $\mathbf{w}$.

Example 5 **Find the area of the triangle with vertices $\mathbf{P} = (1, 2, 3), \mathbf{Q} = (4, 2, 6)$ and $\mathbf{R} = (5, 3, 7)$.**

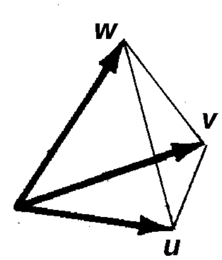
Answer: [Area of the triangle] $= \frac{3}{2}\sqrt{2}$

Example 6 **Calculate the scalar triple product, $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$ for $\mathbf{u} = \langle 3, 3, -1\rangle, \mathbf{v} = \langle 4, 6, 5\rangle$, and $\mathbf{w} = \langle 2, 2, -1\rangle$.**

Answer: $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = -2$



Parallelepiped
FIGURE 1



Tetrahedron
FIGURE 2

Example 7 **What is the volume of the parallelepiped with vertex $\mathbf{P} = (1, 1, 1)$ and adjacent vertices $\mathbf{Q} = (4, 4, 0), \mathbf{R} = (5, 7, 6)$, and $\mathbf{S} = (3, 3, 0)$?**

Answer: [Volume of the parallelepiped] = 2

Example 8 **The vectors $\mathbf{i}, \mathbf{j}$, and $\mathbf{k}$ with their bases at the origin form three edges of a tetrahedron. What is its volume?**

Answer: [Volume] $= \frac{1}{6}$

Interactive Examples

Work the following Interactive Examples on Shenk's web page, http//www.math.ucsd.edu/˜ashenk/:[‡]

Section 12.4: Examples 1–7

[†]Lecture notes to accompany Section 12.4 of *Calculus, Early Transcendentals* by Rogawski.

[‡]The chapter and section numbers on Shenk's web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.