Section 12.5. Planes in three space†

Example 1  
Give an equation of the plane through the point \((2, 3, 4)\) and perpendicular to the vector \((-6, 5, -4)\).

Answer: 
\[-6(x - 2) + 5(y - 3) - 4(z - 4) = 0\]

Example 2  
Give an equation for the plane through \((6, 10, -3)\) and perpendicular to the line \(x = -3t, y = 6 + t, z = 4 - 7t\).

Answer: 
\[-3(x - 6) + (y - 10) - 7(z + 3) = 0\]

Example 3  
Give an equation of the plane through the point \((1, -1, 2)\) that is parallel to the plane \(3x - 5y + 6z = 10\).

Answer: 
\[3(x - 1) - 5(y + 1) + 6(z - 2) = 0\] or (written) \(3x - 5y + 6z = 20\).

Example 4  
Give an equation for the plane through the points \(P = (1, 3, 2), Q = (1, 4, 3),\) and \(R = (2, 5, 0)\).

Answer: 
\[-4(x - 1) + (y - 3) - (z - 2) = 0,\] which simplifies to \(-4x + y - z = -3\)

Example 5  
Check the equation from Example 4 by verifying that the coordinates of \(P = (1, 3, 2), Q = (1, 4, 3),\) and \(R = (2, 5, 0)\) satisfy it.

Answer:  
For \(P = (1, 3, 2): 4x - y + z = 4(1) - 3 + 2 = 3\)  \bullet  
For \(Q = (1, 4, 3): 4x - y + z = 4(1) - 4 + 3 = 3\)  \bullet  
For \(R = (2, 5, 0): 4x - y + z = 4(1) - 4 + 3 = 3\)

Example 6  
Give parametric equations of the line \(L\) through \((3, -4, 5)\) and perpendicular to the plane \(3x - 2y = 5\).

Answer: 
\(L: x = 3 + 3t, y = -4 - 2t, z = 5\)

Interactive Examples

Work the following Interactive Examples on Shenk’s web page, http://www.math.ucsd.edu/~asenk/‡:

Section 12.5: Examples 3–5 and 6–9

†Lecture notes to accompany Section 12.5 of Calculus, Early Transcendentals by Rogawski.

‡The chapter and section numbers on Shenk’s web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.